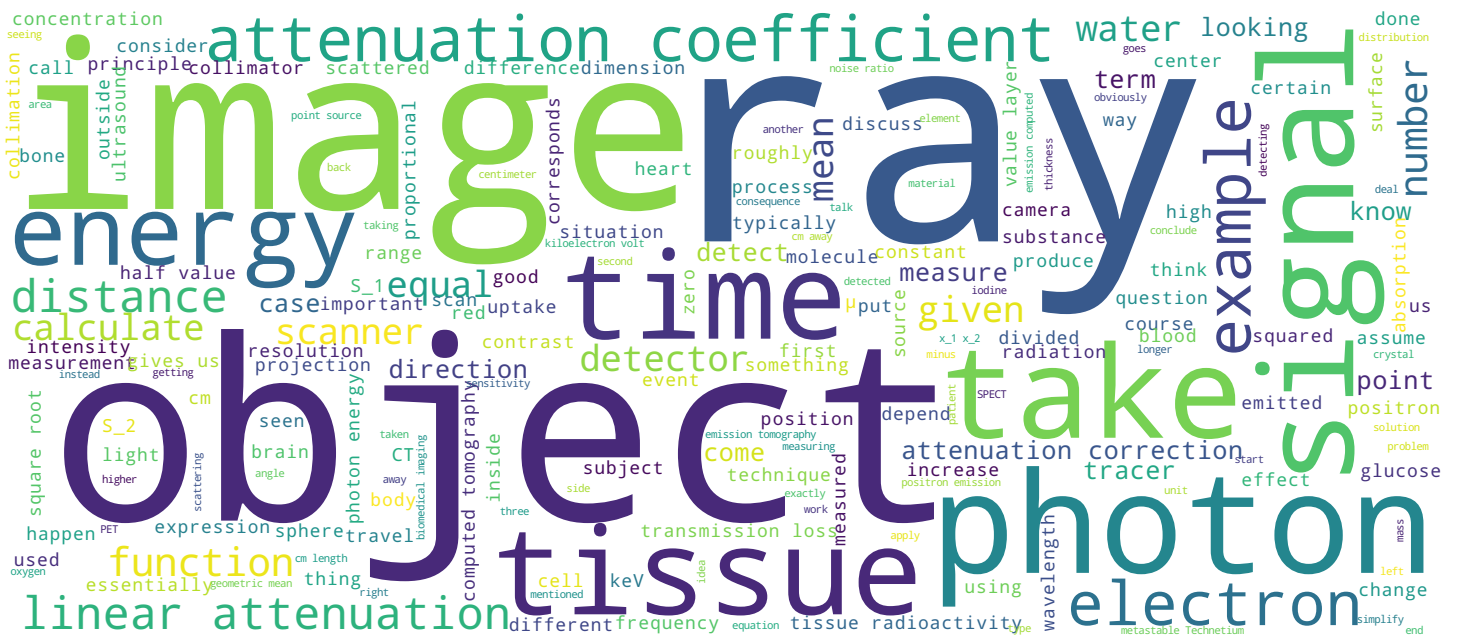


5.3 Attenuation correction

Fundamentals of Biomedical Imaging

Prof. Rolf Gruetter



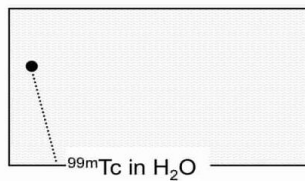
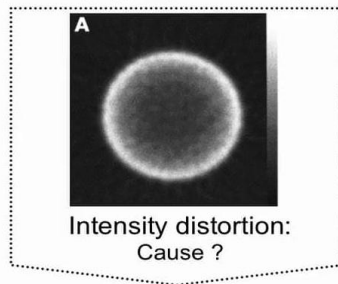
Search MOOC



Video



Signal measured from a homogeneous sphere ($C_T(x,y)=\text{constant}$)



5-8

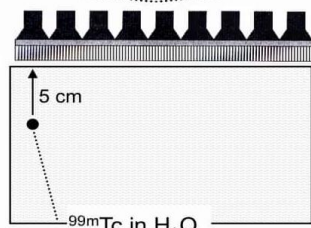
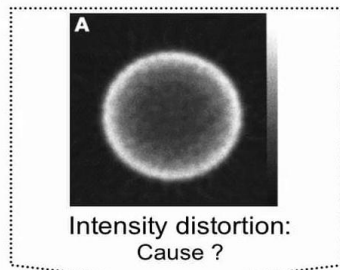
So we have dealt with Collimation, now I want to deal with the second effect that we have to deal with, and that is the attenuation of the emitted x-rays. We have mentioned that this is one of the things that we'd ideally not have. And now what we are going to look at, is an image of an object that is homogeneous. So we'll say it's a sphere. It's filled with a liquid that is doped with, say, Technetium-99. If it's just a liquid, it has a constant radioactivity, a constant specific activity in the tissue, yet if we put that sphere into the scanner, this is the image we get. We are getting bright intensity at the surface, and it is dark inside. This is not the tissue radioactivity, C_t , that we'd like to measure. We know in this case, it is constant, it is homogeneous, yet what we're measuring, is something that's bright on the outside and dark on the inside. So what is the cause for this intensity distortion? And I want to now explain this. So, we'll take you have a situation, an abstract situation; we have a bath filled with water and some radioactive substance. We'll look at this point here and we'll assume that this water has a solution of metastable Technetium-99, but it's an aqueous solution that's in this water bath.

Notes

Summary



Signal measured from a homogeneous sphere ($C_T(x,y)=\text{constant}$)



$$\mu_{\text{water}}(140 \text{ keV}) = 0.16 \text{ cm}^{-1}$$

$$\text{HVL} = 0.693/\mu \approx 4.5 \text{ cm}$$



5-8

Now we have the linear attenuation coefficient of water for 140 keV; that's the energy of the photons of this tracer here of metastable Technetium-99m. This linear attenuation coefficient is 0.16 / cm. Here's our detector. So we have here, a collimator, and the camera behind it. We'll come to the other elements of the camera shortly. We introduced here the distance, and we'll say this object, we're looking now at the point-wise, at one point here, that is about 5 cm away from the camera. So we know the linear attenuation coefficient. We have a homogeneous object. And now we can calculate the attenuation. So, this is something that, a linear attenuation coefficient expressed in per cm, 0.16 / cm, is not very intuitive, and so it makes much more sense to understand the numbers that we're going to provide by looking at the half value layer. So, we have seen that the half value layer is 0.7 divided by μ . And in this case, the half value layer is, 4.5 cm for Technetium in water. So if we're 5 cm away, we should essentially have a reduction of the signal, that is, a bit more than half of the signal. And so we'll talk here about the attenuation T , as a function of the distance D .

Notes

Summary

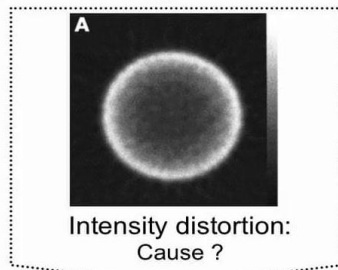


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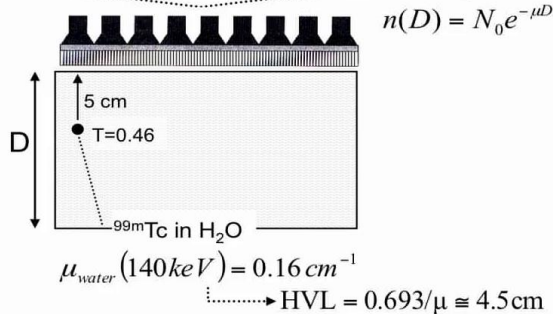
5-3. How to deal with attenuation of the emitted x-rays ?

result of x-ray absorption in tissue

Signal measured from a homogeneous sphere ($C_T(x,y)=\text{constant}$)



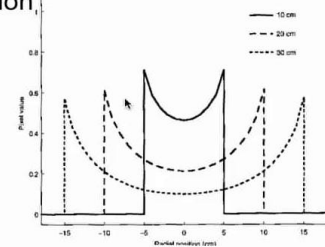
Intensity distortion:
Cause ?



Attenuation T

$$T = \frac{n(D)}{N_0} = e^{-\mu D}$$

1. depends on object dimension
and source location
($D=f(\text{object})$)



5-8

So the distance is how far our object is away from the scanner. Now we've seen from x-rays, that the intensity as a function of distance D , is given by the original intensity incidence x-ray; that's the number of x-rays that are emitted in this direction, N_0 times e to the $-\mu d$. So with this we can calculate the attenuation, and the attenuation is given by the number of photons detected divided by the number of photons emitted in that direction, and that's e to the $-\mu d$. So in this case, the attenuation is 0.46. So we are detecting only 46% of the photons that are emitted in that direction, at a distance of 5 cm. Now, one consequence is, if this is clearly distance dependent, the deeper we are, the more attenuation we have. And so, the attenuation, the reduction of the x-rays that are being detected, depends on the object dimensions, as well as on where the source is. So the distance here, D , where our source is, this part here, is source location and it depends on the size of our object. And here are three examples of an object with 10 cm length, 20 cm length and 30 cm length, and we'll look at the activity detected at the center of the object--it is decreasing with increasing object size.

Notes

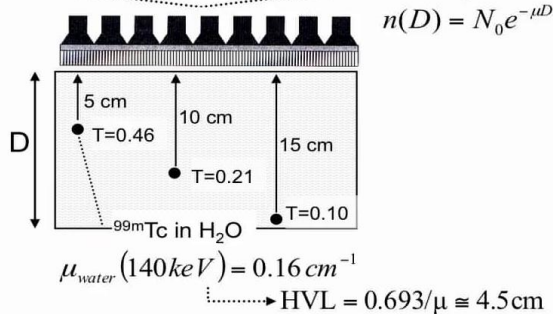
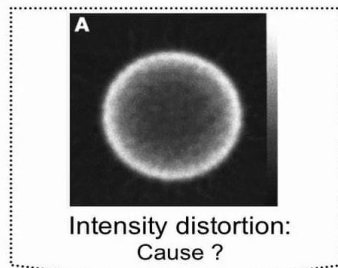
Summary



5-3. How to deal with attenuation of the emitted x-rays ?

result of x-ray absorption in tissue

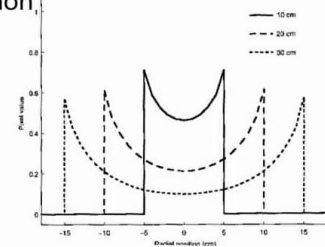
Signal measured from a homogeneous sphere ($C_T(x,y)=\text{constant}$)



Attenuation T

$$T = \frac{n(D)}{N_0} = e^{-\mu D}$$

1. depends on object dimension and source location ($D=f(\text{object})$)



5-8

Because the further away we are at the center from the surface, the longer the distance the x-ray has to travel through the object, and so, the less likely we are to detect an x-ray. So, let's take the next situation here, we're 10 cm away, in this case. So the distance is 10 cm. So we roughly have two times the distance, so it's the square of the attenuation, so 0.46^2 , that is roughly 0.21. So we have 21% of the photons that will survive a travel of 10 cm through water. And the last example here is, 15 cm of a distance, and here, we're only ending up detecting only 10% of the photons. So here we have 5 cm, 10 cm, and here we have 15 cm from the center to the outside. And this is exactly what has happened in here. The substance at the surface of the sphere does not travel much to get to the detector, does not travel much through our water solution, so it is hardly attenuated. Whereas, for the image generation, the substance that's at the center of the sphere, will always have to travel the radius of the sphere. So it's consistently being much more attenuated, so that's why you are seeing an image that instead of being homogeneous, is bright on the edge and dark on the inside.

Notes

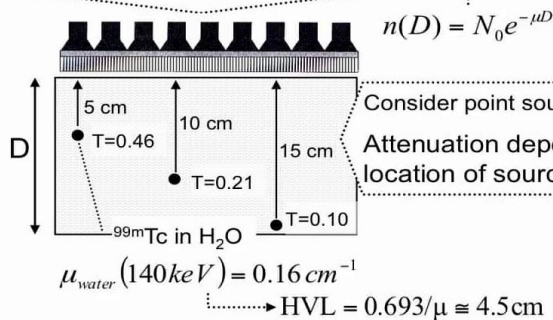
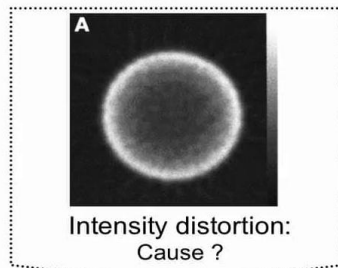
Summary



5-3. How to deal with attenuation of the emitted x-rays ?

result of x-ray absorption in tissue

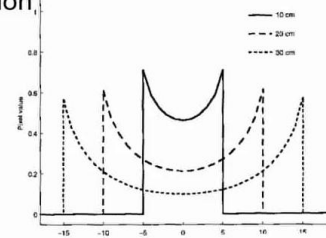
Signal measured from a homogeneous sphere ($C_T(x,y)=\text{constant}$)



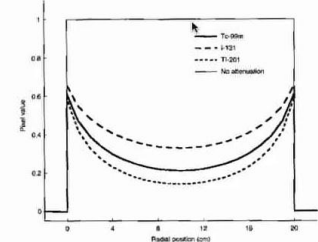
Attenuation T

$$T = \frac{n(D)}{N_0} = e^{-\mu D}$$

1. depends on object dimension and source location ($D=f(\text{object})$)



2. Photon energy $\mu=f(E_\gamma)$



5-8

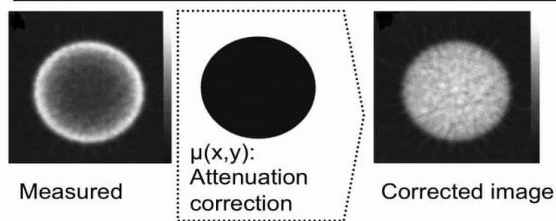
To conclude, if we're looking at the point source, we have taken a homogeneous path, we have looked at three different distances to the camera we're seeing quite clearly that the attenuation depends on the location of the source in the tissue, relative to the camera. There's a second consequence here. and that this the attenuation depends on the specific radioactive substance that we're using on the specific gamma emitter. We have here there situation of-- in a solid line of metastable Technetium-99, we have the dashed line is Iodine-131, and the small dashed line here is Thallium-201. And you can see that there is a difference in attenuation based on the type of isotope. So that is because the linear attenuation coefficient is not uniform for any substance with photon energy, it depends on the photon energy, and so this absorption depends on the tracer that we're using. Of course, here would be the activity we detect if we have no absorption.

Notes

Summary



What are the basic steps in attenuation correction ?



Attenuation correction procedure

- A. Estimated object geometry and estimated $\mu(x,y)$ or measured $\mu(x,y)$
- B. Transmission loss : $T(\text{projection}) = f(\mu(\text{object}), \text{projection})$



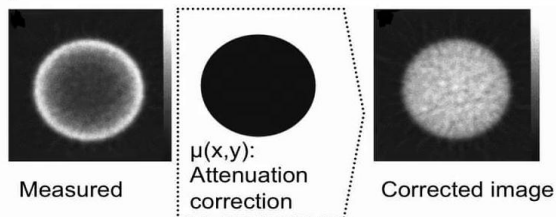
So to conclude, we have two effects: location of the source, dimensions of the object, and type of gamma emitter that we're using. They all contribute in a different way to the attenuation of the x-ray in emission computed tomography. So what are the basic steps in correcting for this distortion in image intensity due to attenuation? What are the basic steps in attenuation correction, that we can do? So here we have the image again. This is what we've measured in our sphere of a homogeneous signal. We will calculate the linear attenuation coefficient, as a function of position. And we can use that for attenuation correction. So we use this. This is the measured, with the measured linear attenuation coefficient or calculated, and we can correct the image intensity. So that is the basic principle of how attenuation correction can in theory be corrected. So how is this procedure-- will this procedure proceed? We'll take an estimated object geometry and an estimated linear attenuation coefficient, or even a measured linear attenuation coefficient. We'll calculate the transmission loss. This transmission loss is a function of the projection. Where are we, how are we, which projection orientation are we measuring?

Notes

Summary



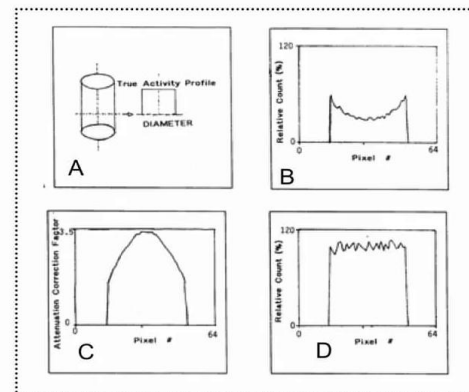
What are the basic steps in attenuation correction ?



Attenuation correction procedure

- Estimated object geometry and estimated $\mu(x,y)$ or measured $\mu(x,y)$
- Transmission loss : $T(\text{projection}) = f(\mu(\text{object}), \text{projection})$
- Attenuation correction $A(x,y) = 1/T(x,y)$
- Corrected $C_{\text{corr}}(x,y) = A(x,y) C(x,y)$

Problem is prior knowledge needed for A (i.e. $\mu(x,y)$)



5-9

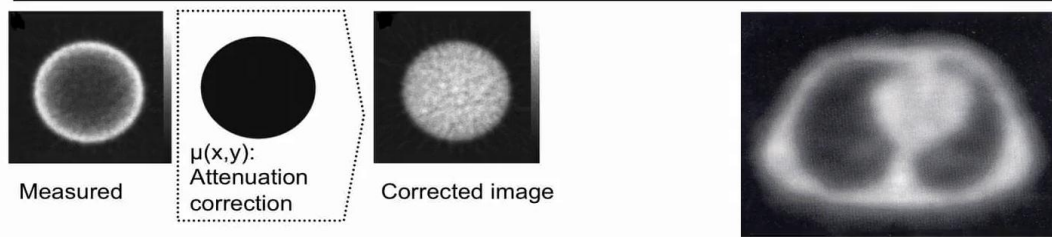
And so it's a function of the distribution of the linear attenuation coefficient over the object. But it's also a function of [inaudible] specific projection that we are registering. So here's our object. We have the measured intensity here and then we can calculate from the transmission loss-- here's the transmission loss characterized. We have the true activity; this gives us the transmission loss, and we can calculate the parameter A as a function of position X and Y. That's one over the transmission loss. That's the attenuation correction. This would look in this example here, like this, so we have to do more correction at the center and less correction at the edges. And then we will calculate the corrected activity profile or image, by multiplying these two terms: the measured activity, C, with the correction parameter, A. And then we get the image here. So this corresponds to this image or this corresponds to the correction calculated based off this. This here corresponds to a cut through this, which is corrected. So that is in principle the attenuation correction procedure that one could apply. However, there is a problem, a practical problem here.

Notes

Summary



What are the basic steps in attenuation correction ?

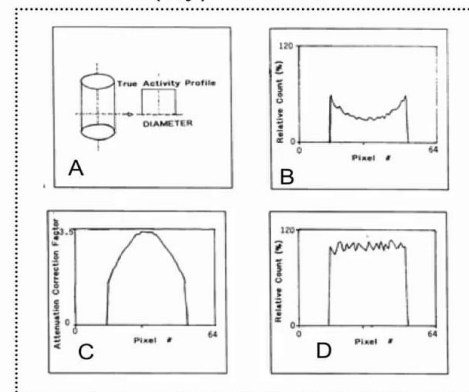


Attenuation correction procedure

- Estimated object geometry and estimated $\mu(x,y)$ or measured $\mu(x,y)$
- Transmission loss : $T(\text{projection}) = f(\mu(\text{object}), \text{projection})$
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Problem is prior knowledge needed for A (i.e. $\mu(x,y)$)

Attenuation correction rarely applied!



5-9

And that is, we need to know the distribution of the linear attenuation coefficient in our subject, in our organs. We also need to know where is the source of the x-ray that we detect. And both of the things are not perfectly achievable. While we can, with for example, computer tomography, get an estimate of the correction matrix, this is the correction matrix of the thorax, we see the heart here, we have the lungs, here's the spine, rib-cage on the outside. We could in theory calculate that. But if we do detect the x-ray in our scanner, we don't know where it came from. We detect an x-ray here. Did it come from here, did it come from here, or did it come from here? On average, we don't know, so it's difficult to apply in practice this attenuation correction. And actually, believe it or not, because of that the attenuation correction is rarely applied, and only in special circumstances, because of these complexities.

Notes

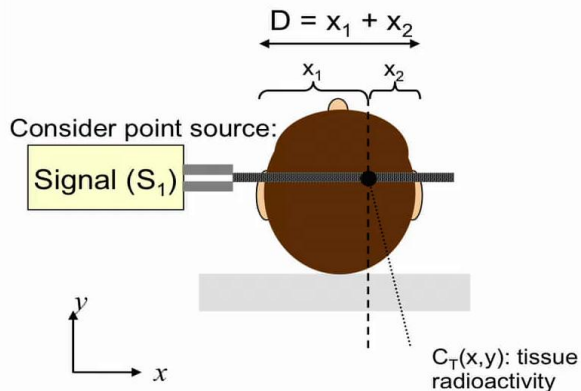
Summary



How to simplify attenuation correction ?

by measuring at 180° using geometric mean

Problem: Spatial dependence of correction



Solution: Geometric mean of the two 180° opposite signals:



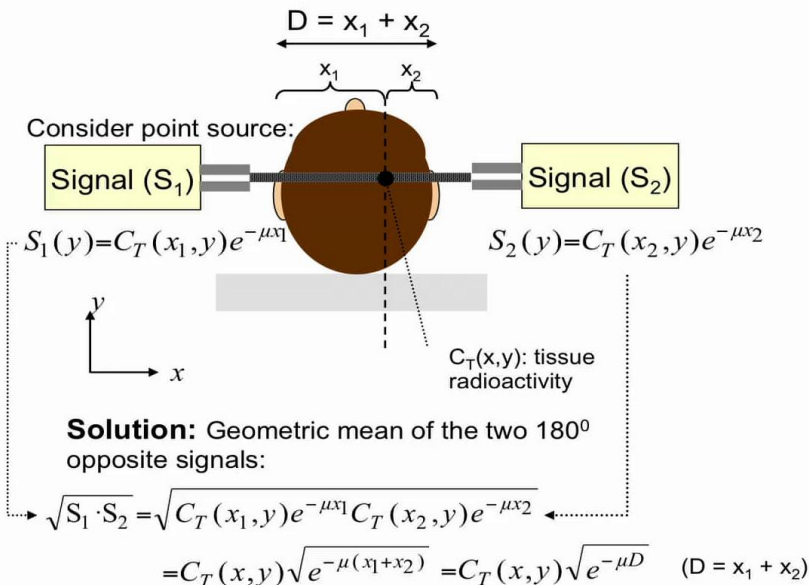
Now I want to discuss one way of how one can simplify the attenuation correction. It is not a generally applicable procedure, but it's a nice introduction to attenuation correction that we'll discuss for the next imaging modality, next week, and that is positron emission tomography. The problem that we have is that, the correction is spatially dependent, it's source dependent. So now we'll consider a point source. Here at this particular point in the subject's head, it's tissue radioactivity, C_T , acquisition x, y , that's the tissue radioactivity. And now, what we will do, the solution to simplify this is, to consider the geometric mean of signals that are acquired at 180° opposite. So I'll first introduce the position of our object. We have the distance from our object to the left end of the subject is x_1 , and distance from our object to the right end of the patient is x_2 . So the dimensions along x of our object at this point is given by $x_1 + x_2$, and we'll call that big D . So now we have the signal S_1 . That's detected on the left side. We can see the perfect collimation here, and we have a signal S_2 , collected from the right side, from this point source.

Notes

Summary



Problem: Spatial dependence of correction



5-10

So I'll give you the explicit expression here: we have the tissue radioactivity $C_T(x, y)$, so that's position x, y , seen from detector 1 times e to the $-\mu x_1$. And we'll assume here for simplicity, a homogeneous linear attenuation coefficient. On the right side, we have signal S_2 , is given by the same tissue radioactivity times e to the $-\mu x_2$. These are really not equal signs, they're proportionality but we'll just for simplicity, we'll forget all the proportionality [inaudible]. So now what can we do? We'll take signal S_1 and S_2 , multiply them by each other and take the square root. That's what we've done here. We'll take the expression on the left. This one. Multiply it by the expression on the right. This one. And take the square root of that. The C_T , the tissue radioactivity, we can take that out of the root, because that's in there squared. And then we have e to the $-\mu x_1 + x_2$. And that is equal to the square root of e to the $-\mu D$. A reminder here, D is equal to $x_1 + x_2$.

Notes

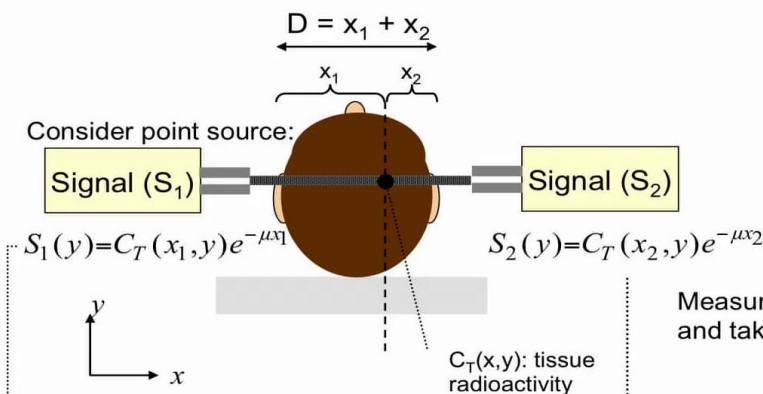
Summary



How to simplify attenuation correction ?

by measuring at 180° using geometric mean

Problem: Spatial dependence of correction



$$\sqrt{S_1 \cdot S_2} = C_T(x, y)e^{-\mu D / 2}$$

NB. This correction can be used in emission tomography for focal uptake (i.e. uptake limited to a specific region)

Measure at 180° simultaneously and take the geometric mean

Solution: Geometric mean of the two 180° opposite signals:

$$\begin{aligned} \sqrt{S_1 \cdot S_2} &= \sqrt{C_T(x_1, y)e^{-\mu x_1} C_T(x_2, y)e^{-\mu x_2}} \\ &= C_T(x, y) \sqrt{e^{-\mu(x_1 + x_2)}} = C_T(x, y) \sqrt{e^{-\mu D}} \quad (D = x_1 + x_2) \end{aligned}$$

5-10

So, if we now simplify that, then we have the geometric mean, that is the multiplication of S_1 with S_2 , taking the square root that's equal to the tissue radioactivity times e to the $-\mu D$ divided by 2. So this now means that if we correct our signal this way, from our subject, we no longer have to be concerned from where exactly the x-ray in the body is emitted, but we only have to consider with the total travel distance of the x-ray, through the object alone, the left and the right, the total travel distance passing through the object. This correction works in this example, for a point-wide source. It works in reality, if the uptake is limited to a specific region, so focal uptake. Not if the uptake is diffused over the entire body; it will not work as nicely as if we're looking at as focal uptake. And one example here are the heart scans, with imaging computed tomography, here the heart has an intense uptake and the rest of the body surrounding the heart does not have such an intense uptake. To summarize, we mentioned at 180 degrees opposite the signal S_1 , S_2 , simultaneously multiply them and basically take the geometric mean.

Notes

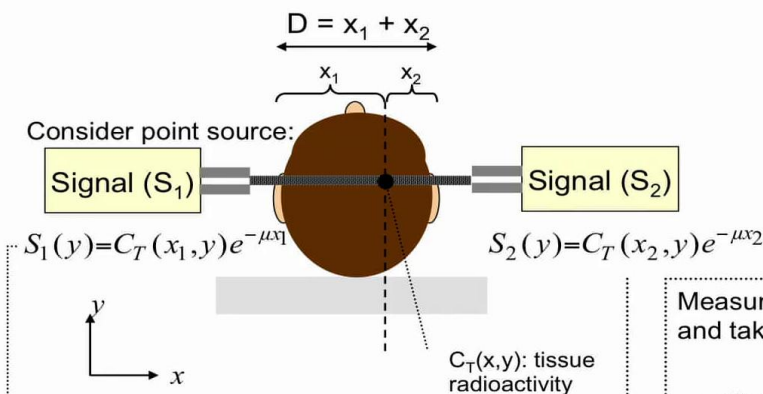
Summary



How to simplify attenuation correction ?

by measuring at 180° using geometric mean

Problem: Spatial dependence of correction



Solution: Geometric mean of the two 180° opposite signals:

$$\begin{aligned} \sqrt{S_1 \cdot S_2} &= \sqrt{C_T(x_1, y)e^{-\mu x_1} C_T(x_2, y)e^{-\mu x_2}} \\ &= C_T(x, y) \sqrt{e^{-\mu(x_1 + x_2)}} = C_T(x, y) \sqrt{e^{-\mu D}} \quad (D = x_1 + x_2) \end{aligned}$$

$$\sqrt{S_1 \cdot S_2} = C_T(x, y)e^{-\mu D/2}$$

NB. This correction can be used in emission tomography for focal uptake (i.e. uptake limited to a specific region)

Measure at 180° simultaneously and take the geometric mean

→ attenuation correction depends only on dimension of object along the measured Radon transform

5-10

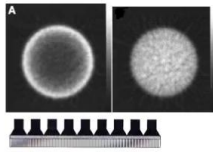
And then that means that the attenuation correction that we have to apply to the signal, for each Radon, for each projection, depends only on the dimension of the object, along the measured Radon transfer. That is, if you will, the thickness of the object, D , in this case.

Notes

Summary

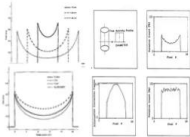


15m 45s



Determination of the Attenuation Map in Emission Tomography

Fig. 3 from Habib Zaidi et al. J. Nucl. Med. 2003; 44:291



The Essential Physics for Medical Imaging

Jerrold T. Bushberg



Unidentifiable source (from the internet)

So this is a simplification, but clearly if we do a scan in this direction, we have the scanner here or if we have the scanner here, the distance that the x-rays in total pass through, is different for this dimension and this dimension. At least for most of us.

Notes

Summary

16m 05s

