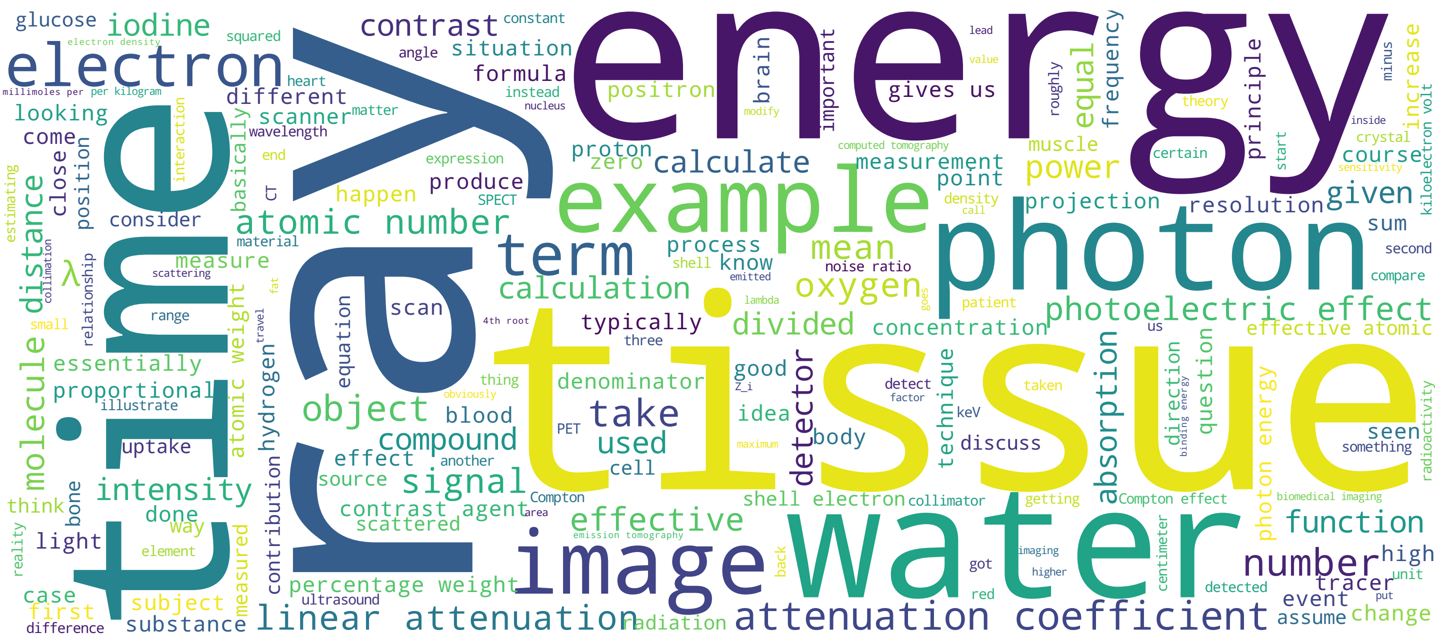
3.4 Tissue dependence of x-ray absorption

Fundamentals of Biomedical Imaging

Prof. Rolf Gruetter



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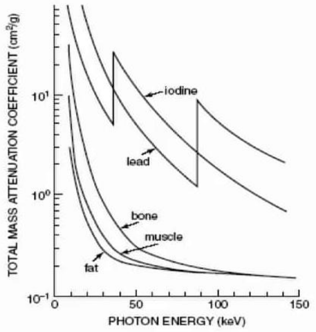
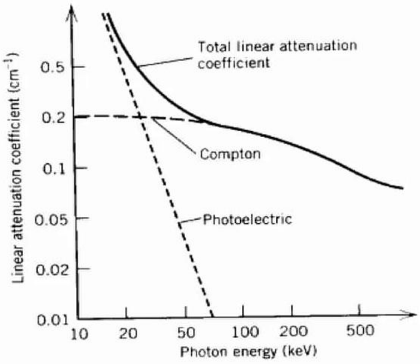
Video

3-4. What affects the linear attenuation coefficient ?

effective Z, Z_{eff}





Linear attenuation coefficient of water and the contribution of each interaction to the total attenuation of X-rays as a function of energy.

Compton:

depends mainly on the e density ρ_e

modestly on energy of x-rays E_ν

3-11

In this section, we are going to address the question on how is the linear attenuation coefficient dependent on the tissue. So to recap, we have the attenuation coefficient here it's a mass attenuation coefficient for iodine and lead. We see the absorption edges, we have them twice for the different binding energies. This is the k-shell, this is the l-shell here. We have bone, muscle, and fat are the acting mass attenuation coefficients. And now, if we just focus on Compton and photoelectric effect, then we can see that in tissue, the total linear attenuation coefficient can be seen as a relationship from a contribution from the photoelectric effect, certainly at the low energies, and at the higher energies it's the Compton effect that prevails. But what's important here is that the linear attenuation coefficient in this case of water, is the contribution of the x-rays from Compton and photoelectric effect and therefore energy dependent. Now let's look at Compton. Compton mainly depends on the number of outer shell electrons per volume, per [inaudible] present, so it mainly depends on the electron density ρ. Therefore, it is only modestly dependent on the energy of the x-rays.

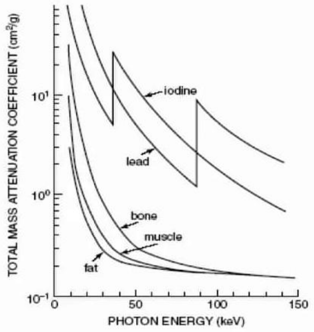
Notes

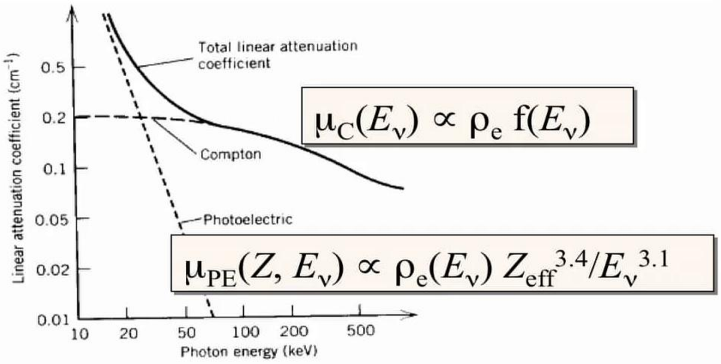
Summary

0m 04s

3-4. What affects the linear attenuation coefficient ?

effective Z, Z_{eff}





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Photoelectric effect:

depends strongly on atomic number Z

Biology:

empirical Z_{eff}

3-11

[inaudible] here, and this we can see here, notice that we have both scales, again, logarithmic so this is a very modest dependence with photon energy. So we can say the linear attenuation coefficient due to the Compton effect as a function of energy is proportional, essentially to the density of the electrons and then some smooth function as a function of energy. The photoelectric effect, on the other hand depends strongly on the atomic number Z. This we can see here, for iodine and lead for example, it has a strong effect for the two. So, this means for the Compton effect we have for the photoelectric effect for biology we will look at an empirical effective atomic number to calculate because we are typically dealing with tissues which are a mixture of different atoms and so different atomic numbers Z. And we have to deal with these different contributions. So, the photoelectric effect-- this is from theory here-- is proportional to the density of the electrons. The density of the electrons depends on the energy of the photon. Remember, if we don't have enough energy to ionize a k-shell then all the k-shell electrons are not seen by the x-ray so therefore the electron density that can participate in the reaction is dependent on the energy of the photon.

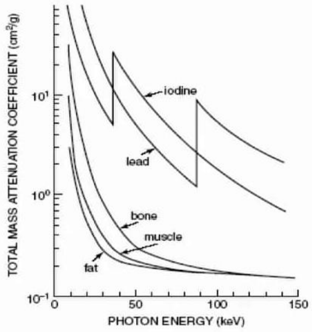
Notes

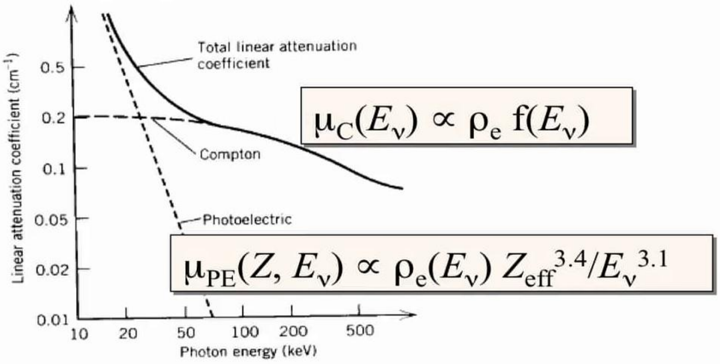
Summary

1m 26s

3-4. What affects the linear attenuation coefficient ?

effective Z, Z_{eff}





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Compton:

- depends mainly on the e density ρ_e
- modestly on energy of x-rays E_ν

Photoelectric effect:

- depends strongly on atomic number Z

Biology:

empirical Z_{eff}

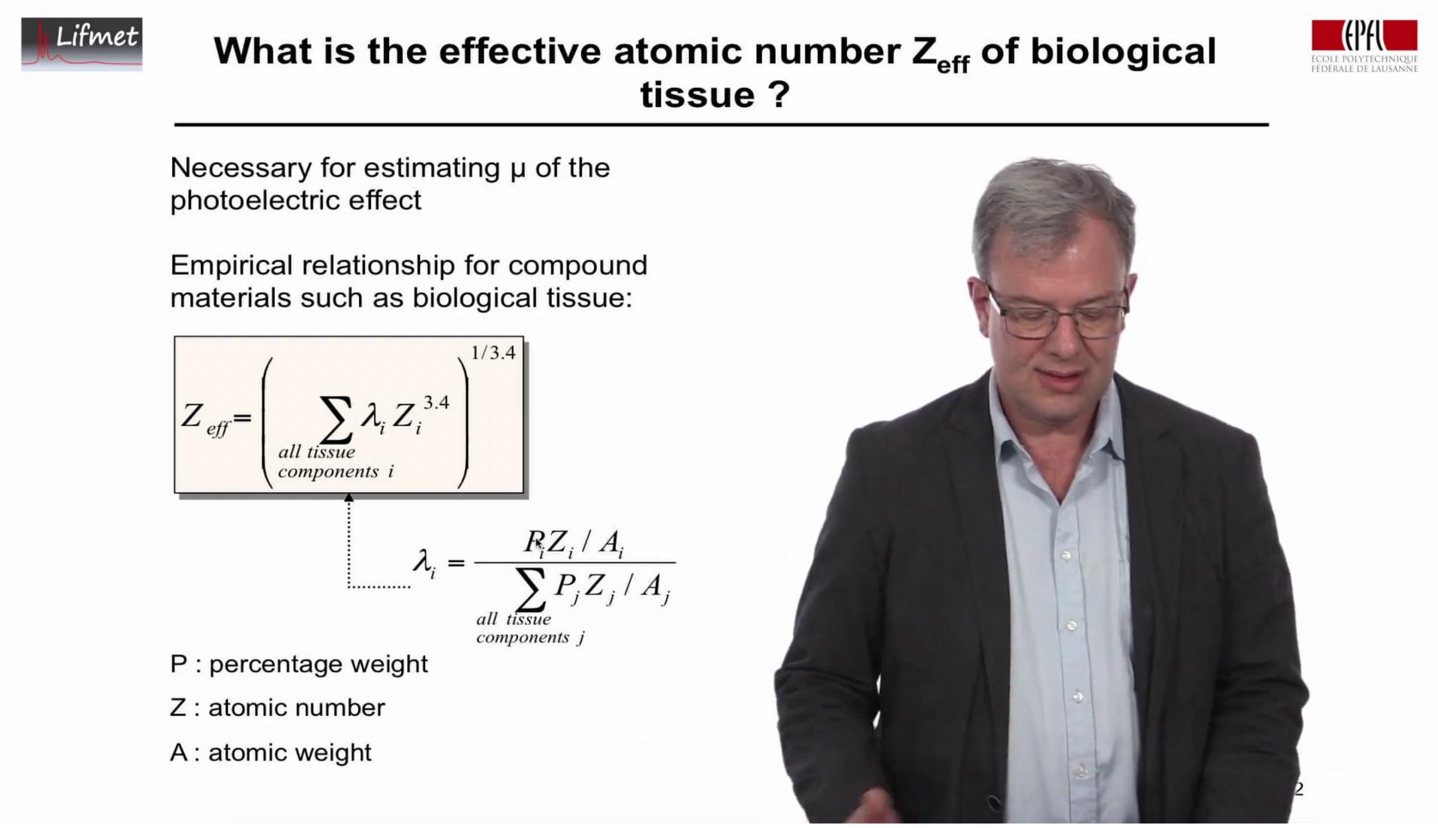
3-11

The photoelectric effect here, according to theory is proportional to the effective set atomic number to the power of 3.4, and inversely proportional to the energy of the photon to the power of roughly 3. It decreases rapidly, and this we can understand from Compton mechanical considerations. I'll just give a plausibility argument here that it is a match... if the match of the energy of the photon corresponds very closely to the binding energy then the interaction is greatest.

Notes

Summary

3m 01s



Now I want to provide an empirical calculation for the effective atomic number of biological tissue. We need this to have an idea for estimating the linear attenuation coefficient of the photoelectric effect. And we'll start with en empirical relationship. I've taken this from the textbook Cho. I have not found a rigorous demonstration of this but we'll use this here as ground truth that the calculation of the effective atomic number follows this equation here. It is very useful so that we can understand certain properties of tissues and also how we can modify the atomic number. So basically what this equation says here: the atomic number, the effective atomic number is equal to the 3.4th root of this sum here. This sum here is summed over all tissue components to indicate by the index i with a weighting factor λ_i times the atomic number Z_i to the power of 3.4. The weighting coefficient λ here is given by this expression here where we have P is the percentage weight of a compound i. Z is the atomic number of the compound i and A is the atomic weight of compound i.

Notes

Summary

3m 33s

What is the effective atomic number Z_{eff} of biological tissue ?

Necessary for estimating μ of the photoelectric effect

Empirical relationship for compound materials such as biological tissue:

$$Z_{eff} = \left(\sum_{\substack{all\ tissue \\ components\ i}} \lambda_i Z_i^{3.4} \right)^{1/3.4}$$

$$\lambda_i = \frac{P_i Z_i / A_i}{\sum_{\substack{all\ tissue \\ components\ j}} P_j Z_j / A_j}$$

P : percentage weight

Z : atomic number

A : atomic weight

Example: Estimation of Z_{eff} for water

^{1}H: Z=A=1, P=11%,

^{16}O: Z=8, A=16, P=89%

Denominator of λ : $\sum P_j \frac{Z_j}{A_j} = 11\frac{1}{1} + 89\frac{8}{16} = 55.5$

Protons: λ = 11/55.5=0.20

Oxygen: λ = 44.5/55.5=0.80.

3-12

So it's this term here, divided by the sum of all components: tissue weight, percentage weight, atomic number and atomic weight, divided by atomic weight for all the components that are considered in our tissue. So let me illustrate this calculation, this formula for estimating the effective atomic number for water. For hydrogen, we have Z, we have one proton, so Z is 1. We have only one proton, so the atomic weight is 1. And the percentage weight is 11%. We have two protons and one oxygen in water. So let's look at the oxygen's extreme. Z is 8, so we have eight protons, we have eight neutrons in the nucleus so the atomic weight is 16, and the percentage weight is 89%. Total weight of a water molecule is 18, so 2/18 is 1/9, that's 11% and 16/18, that's 8/9, that is 89%. So, for the denominator, that is this term here we'll calculate this explicitly here, here's the calculation. We have Z/A for hydrogen, Z/A for oxygen, we have percentage weight and we get the denominator is 55.5. So the *λ* for protons, *λ_i*, *λ_1* here would be 11 / 55.5 and that is essentially 0.20. For the oxygen, the *λ* is 44.5 divided by 55.5, so it's *P_i* times *Z_i* over *A_i*, so it's *P* divided by two, so this is 44.5 divided by 55.5, this is 0.8.

Notes

Summary

4m 55s

What is the effective atomic number Z_{eff} of biological tissue ?

Necessary for estimating μ of the photoelectric effect

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$$Z_{eff} = \left(\sum_{\substack{\text{all tissue} \\ \text{components } i}} \lambda_i Z_i^{3.4} \right)^{1/3.4}$$

$$\lambda_i = \frac{P_i Z_i / A_i}{\sum_{\substack{\text{all tissue} \\ \text{components } j}} P_j Z_j / A_j}$$

P : percentage weight

Z : atomic number

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Protons: λ = 11/55.5=0.20

Oxygen: λ = 44.5/55.5=0.80.

$Z_{eff}=(0.2 \cdot 1^{3.4}+0.8 \cdot 8^{3.4})^{1/3.4}$

$=(0.2+1180 \cdot 0.8)^{1/3.4}$

$=944^{1/3.4}=7.5$

How good were we ?

3-12

This is the weighting factor λ for oxygen. And with that, we've got the two lambdas, we've got these terms here so this is for oxygen, 20% is the percentage weight. One is the Z_i over A_i for hydrogen, that's 1. We have here, this term, and then we have 0.8 times this term times Z to the power of 3.4, so 0.8 is the λ. This is the Z to 3.4, this is the Z to 3.4 for hydrogen. This is 0.2 plus this term here, and we take the 3.4th root. All this is 944 to the 3.4th root, and we get a Z effective of 7.5. Now this makes sense, we have a big contribution from oxygen. Oxygen has a set of eight, that the Z is close to oxygen, that makes sense. So, question, How good were we? Does this have any resemblance with reality?

Notes

Summary

6m 46s

What is the % mass composition P_i of select biological tissues ?

% Composition (by Mass)	*Adipose Tissue*	*Muscle (Striated)*	*Water*	*Bone (Femur)*
Hydrogen	11.2	10.2	11.2	8.4
Carbon	57.3	12.3		27.6
Nitrogen	1.1	3.5		2.7
Oxygen	30.3	72.9	88.8	41.0
Sodium		0.08		
Magnesium		0.02		7.0
Phosphorus		0.2		7.0
Sulfur	0.06	0.5		0.2
Potassium		0.3		
Calcium		0.007		14.7
	Z_{eff} ~ 6	7.4		12

Carbon:

Z/A=6/12
(same for O, 8/16)

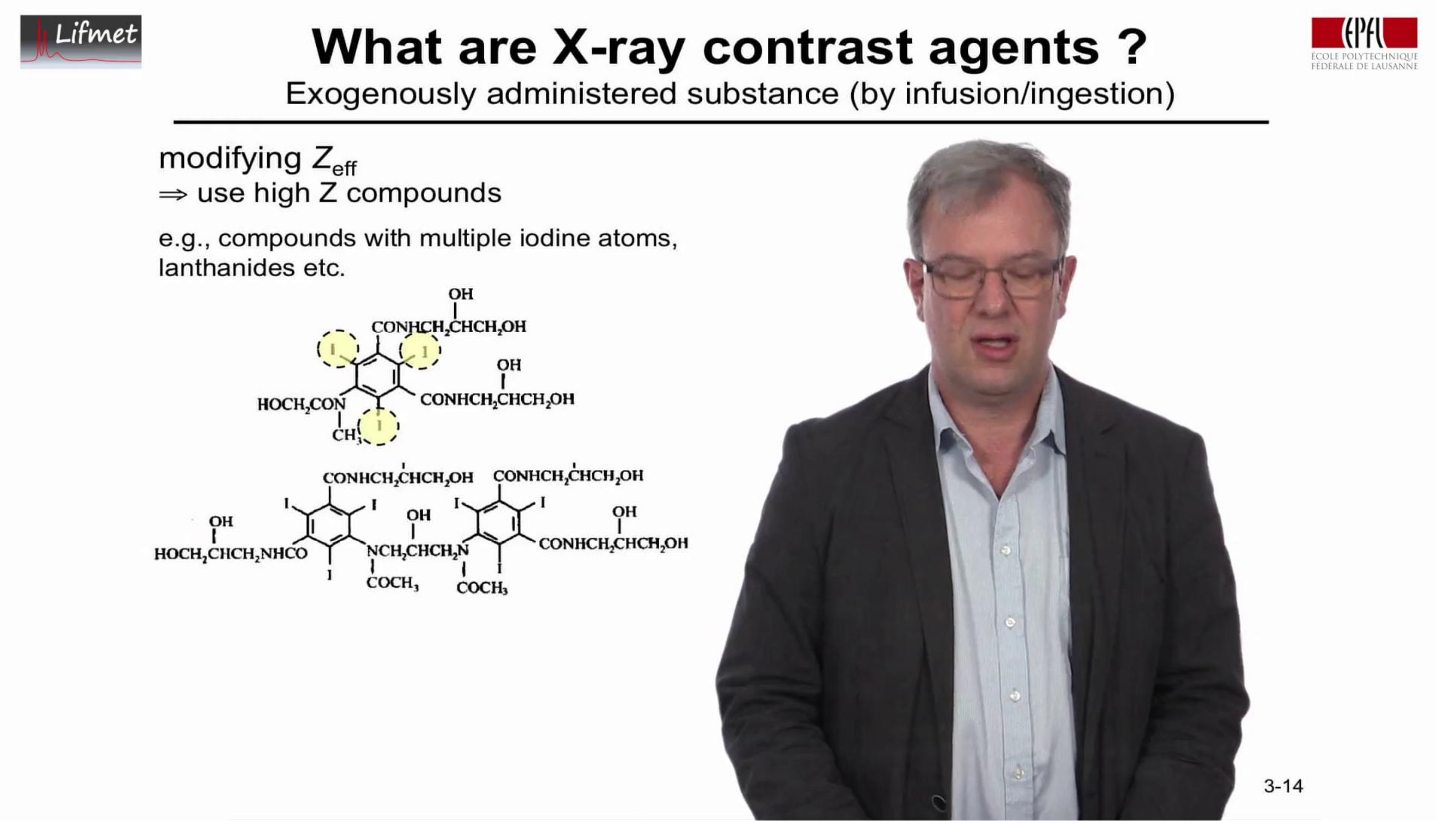
3-13

So we will look at some select biological tissues the percentage mass composition. So we have all the elements here, percentage mass commposition hydrogen, carbon, nitrogen, etc., oxygen, and so on. This is for fat, this is for muscle, here's water, and we have the femur bone as an example. Water is what we calculated, and we want to compare that to some experimental values, but we do not have exactly the values for water, so we'll compare it to muscle. And we have found these values in the literature. Z** effective for adipose tissue was determined to be 6. For striated muscle it was 7.4, and for bone it was 12. And here, we can compare the muscle with water because the carbon has a *Z* to *A* of 6/12, oxygen is 8/16, so it is 6/12, so we get *Z* to *A* of 6/12. It is very similar to oxygen, and so this is a good approximation of what we have for water. So actually it turns out that the 7.5 that we calculated based on this empirical formula is very close to the reality as seen in the muscle here.

Notes

Summary

7m 50s



So now I've shown you the calculation, I've given you the formula for the calculation of the effective Z, and the issue behind this is that we would like sometimes to be in a situation to alter the contrast of the tissue. We would like to inject, for example, a substance into the bloodstream to depict the vessel structure in the patient or in the organism of interest and so we would like to see that the substance that we inject alters the absorption of the x-rays. And these substances that alter the contrast they are called contrast agents. And they're typically administered exogenously that is, they don't exist in our bodies, typically by infusion, but sometimes also you swallow the compound. For x-ray, they are compounds that modify the effective *Z*. So, what is needed for a contrast agent for x-ray imaging is a compound with high *Z*. So, typically what is being used are compounds with multiple iodine atoms sometimes also used with lanthanides. Here's an example of a molecule. We have one iodine, we have the second iodine and we have the third iodine here. So we have three iodines on this molecule. Here's another example of x-ray contrast agents.

Notes

Summary

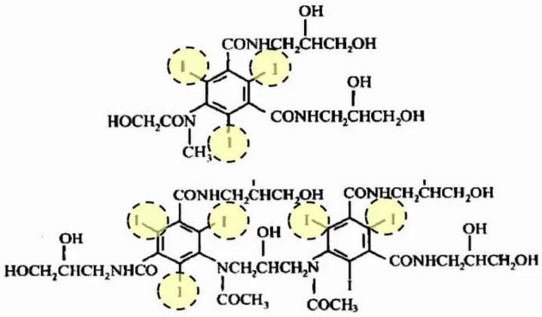
9m 11s

What are X-ray contrast agents ?

Exogenously administered substance (by infusion/ingestion)

modifying Z_{eff}
⇒ use high Z compounds

e.g., compounds with multiple iodine atoms, lanthanides etc.



Z_{eff} of (water+10 mmol/kg iodine) = ?

Iodine:
P_I = 10[mmol/kg]×127[mg/mmol] = 0.127%
Z_I = 53
A_I = 127

Calculation of Z_{eff}:

3-14

We have the iodine here, here, here, here, and here. And so we have multiple iodine atoms that are being absorbed, and actually we can see here there's one iodine that we forgot to highlight here. So there's six iodines in this molecule that are present. So, the idea is we inject a foreign compound that carries as many heavy *Z* atoms on it so it has, for a given concentration of the compound it has an alteration of the effective *Z*. And so here the question is, let's say we have water and we add 10 millimoles per kilogram of iodine to the water. And we want to know how much is this effective *Z* altered. So let's look at iodine. We have seen already the pertinent numbers for water. Iodine in this case has a percentage weight of 0.127%. It's calculated by the concentration of 10 millimoles per kilogram. For water, a kilogram is a liter. And the molecular weight is 127 milligrams per millimole. *Z* for iodine is 53, and the atomic weight is 127. *Z* is the number of protons, *A* is the number of protons and neutrons. So now, let's do the calculation for *Z* effective. We'll, for recapitulation, we'll reproduce the formulas here.

Notes

Summary

10m 32s

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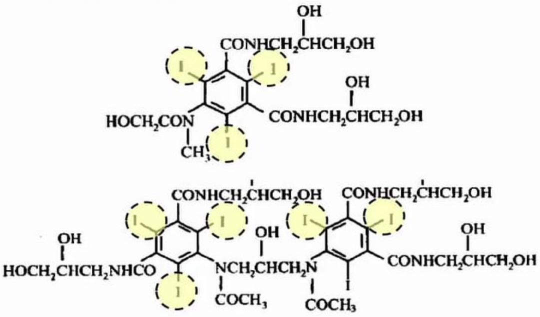

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$$Z_{eff} = \left(\sum_{\substack{all\ tissue \\ components\ i}} \lambda_i Z_i^{3.4} \right)^{1/3.4} \qquad \lambda_i = \frac{P_i Z_i / A_i}{\sum_{\substack{all\ tissue \\ components\ j}} P_j Z_j / A_j}$$

~ denominator λ of pure H_2O

Denominator of λ :

$$\sum P_i \frac{Z_i}{A_i} = 11\frac{1}{1} + 89\frac{8}{16} + 0.13\frac{53}{127} = 55.6$$

→ λ of H and O are as for water:

λ_H=0.2

λ_O=0.8

$$\lambda_I = \frac{0.127 \cdot 53/127}{55.6} = \frac{0.053}{55.6} = 9.5 \times 10^{-4}$$

$$Z_{eff}^{3.4} = 0.2 \cdot 1^{3.4} + 0.8 \cdot 8^{3.4} + 9.5 \cdot 10^{-4} \cdot 53^{3.4}$$

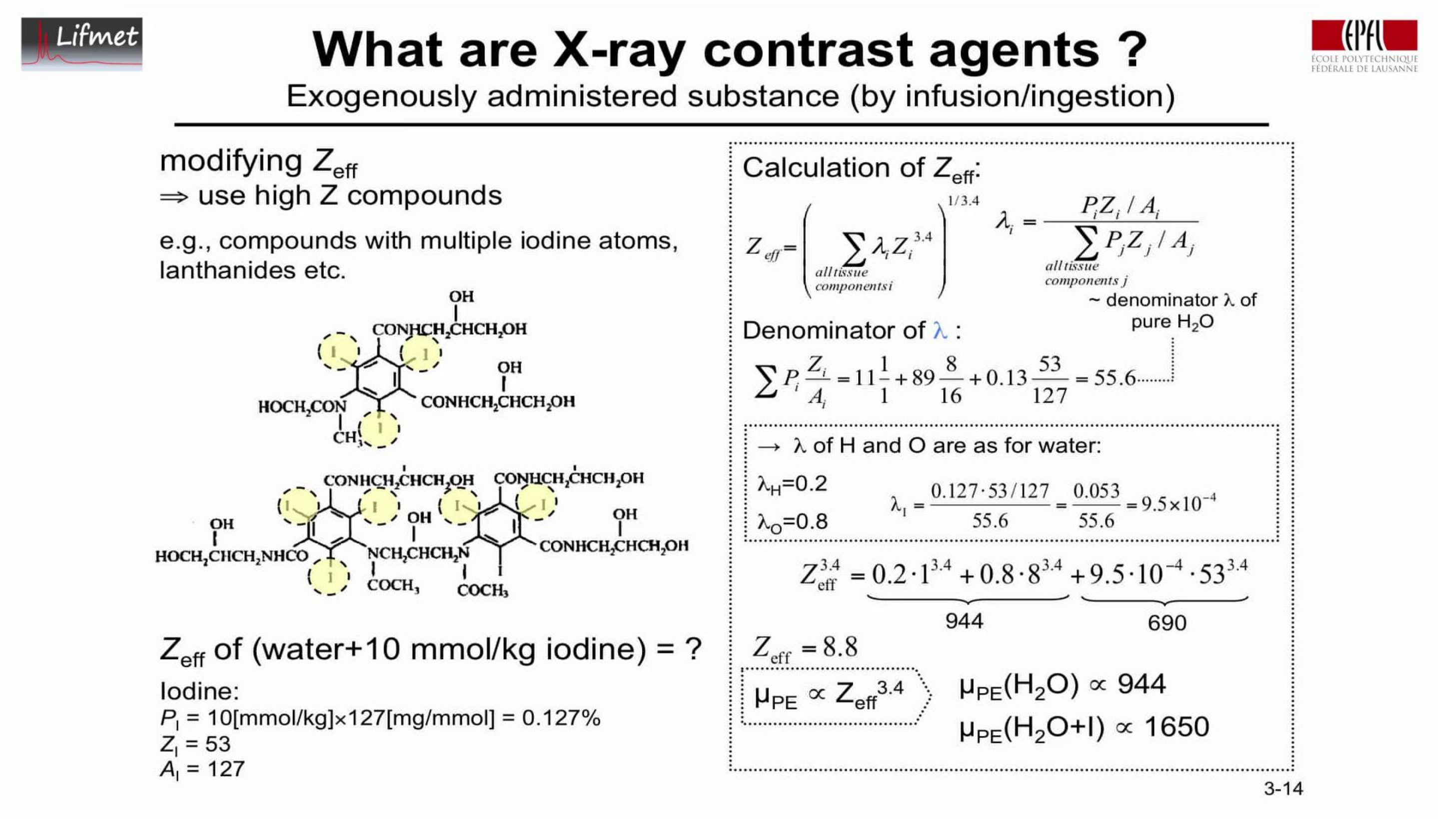
3-14

So here's the formula for *Z* effective. We have the calculation of the lambdas. The denominator of *λ* is contribution of hydrogen, contribution of oxygen, those are the same terms and the contribution of iodine, and instead of 55.5 we get 55.6. So, this is essentially the denominator of pure H2O down to the third significant digit so we can assume that this is the same number. This has not changed the denominator and it's this sum that is here. So that means that the *λ* for hydrogen and oxygen are as for water. And we have the *λ* for hydrogen of 0.2, and the λ for oxygen for 0.8 and now we need to calculate the *λ* for iodine. So, for the *λ* we have the denominator is 55.6 we have the percentage weight here, it's 0.127% we have *Z* divided by *A*, this gives us this term, and we obtain this tiny number here, which is basically one per mill. It's 9.5 times 10 to the power of -4, this is 0.001, so it's 1 per mill, or 0.1%. That's the *λ* here, so it's much smaller. But, if we calculate Z effective to the power of 3.4, so I take this formula to the power of 3.4 so I just calculate this term here.

Notes

Summary

12m 07s



Then we have to remember that we multiply *Z* to the power of 3.4 times the lambdas, and so even though this number is small we have a big number here to the power of 3.4. This is for water, 944, this number we get here and for iodine it's 690. So in conclusion, the *Z* effective for 10 millimolar iodine added to the water solution, the *Z* effective changes from 7.5 to 8.8. That seems like a fairly modest change. So, we'll say, well, the photoelectric effect linear attenuation coefficient is essentially proportional to the effective *Z* to the power of 3.4. And if we take this calculation we see, we have this term here, for water it's 944, pure water, and for water plus 10 millimolar iodine we have this term here, is, this is proportional to 1,650. It doesn't matter what the units are for these numbers, 944 and 1,650. What matters is the relative term here. So the ratio of these two essentially gives us the change in linear attenuation coefficient for the photoelectric effect at low photon energies. And you can see if we're very generous in estimating this we actually see that 1,650 is close to twice as big as 900, so it's almost a two-fold increase in absorption of the x-rays.

Notes

Summary

13m 31s

What are X-ray contrast agents ?

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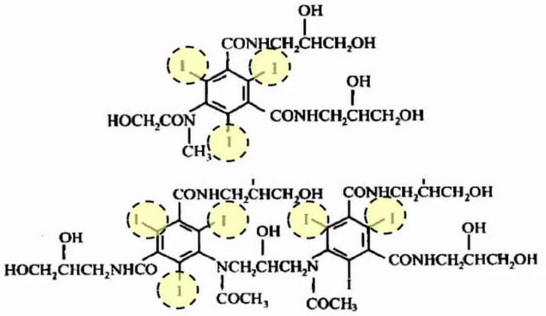

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~ denominator λ of pure H_2O

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$$\lambda_I = \frac{0.127 \cdot 53/127}{55.6} = \frac{0.053}{55.6} = 9.5\times10^{-4}$$

$$Z_{eff}^{3.4} = \underbrace{0.2\cdot1^{3.4} + 0.8\cdot8^{3.4}}_{944} + \underbrace{9.5\cdot10^{-4}\cdot53^{3.4}}_{690}$$

$Z_{eff} = 8.8$

$\mu_{PE} \propto Z_{eff}^{3.4}$ $\quad \mu_{PE}(H_2O) \propto 944$
$\mu_{PE}(H_2O\text{+}I) \propto 1650$

3-14

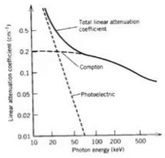
So imagine this. We have this molecule here which is three iodines, so this means actually three millimoles per kilogram of this molecule in water will increase the absorption by nearly a factor of two. So that's a very potent way of altering the absorption of x-rays in tissue in the presence of these agents, the presence of these molecules that are called contrast agents, so they modify the contrast.

Notes

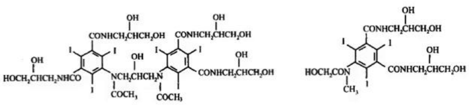
Summary

15m 04s

Credits



“Foundations of Medical Imaging”
Fig. 6-1, from Zhang-Hee Cho, Joie J. Jones, Manbir Singh



“Introduction to biomedical imaging”
Fig. 1-18, from Andrew Webb

% Composition (by Mass)	*Adipose Tissue*	*Muscle (Striated)*	*Water*	*Bone (Femur)*
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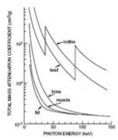

Unidentifiable sources

And in the problem set, we give you a similar calculation to do but instead of using iodine we're going to use gadolinium. Gadolinium is interesting because it has similar effects on the x-ray image, but it is also used used as a contrast agent in MRI, which we will discuss later this semester.

Notes

Summary

15m 32s