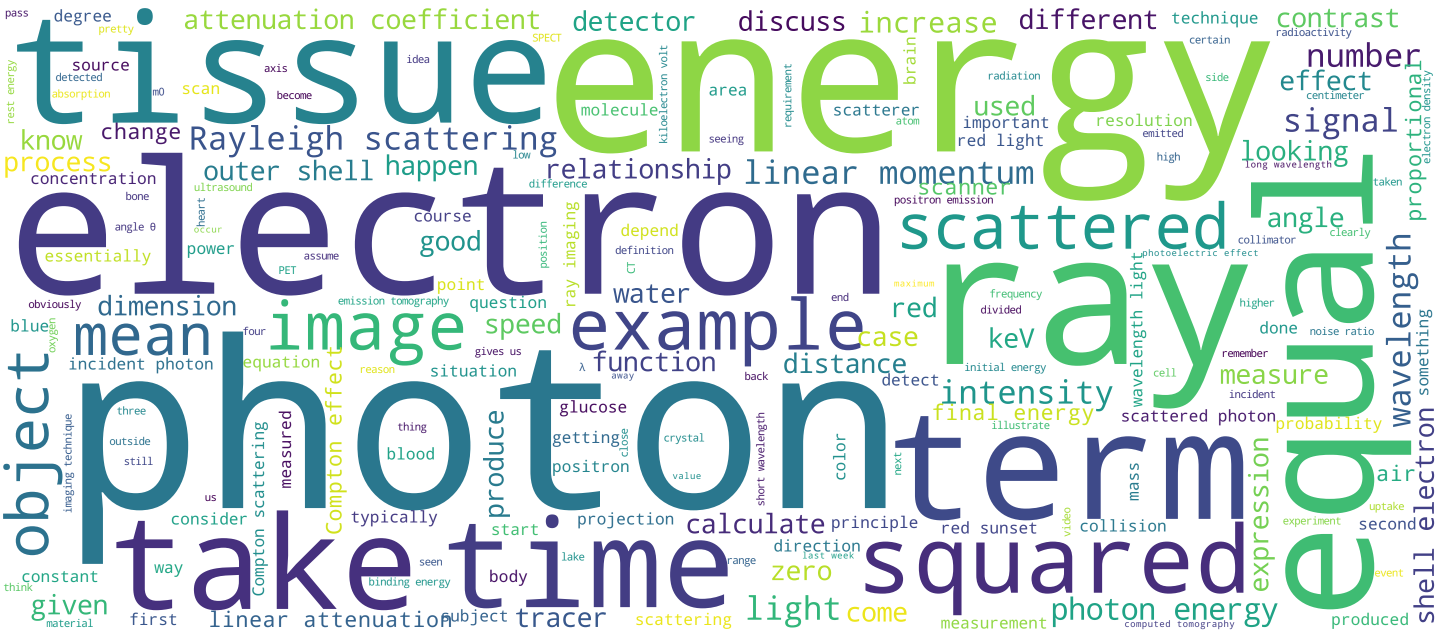
3.2 interactions of x-rays with tissue
(Rayleigh & Compton)

Fundamentals of Biomedical Imaging

Prof. Rolf Gruetter



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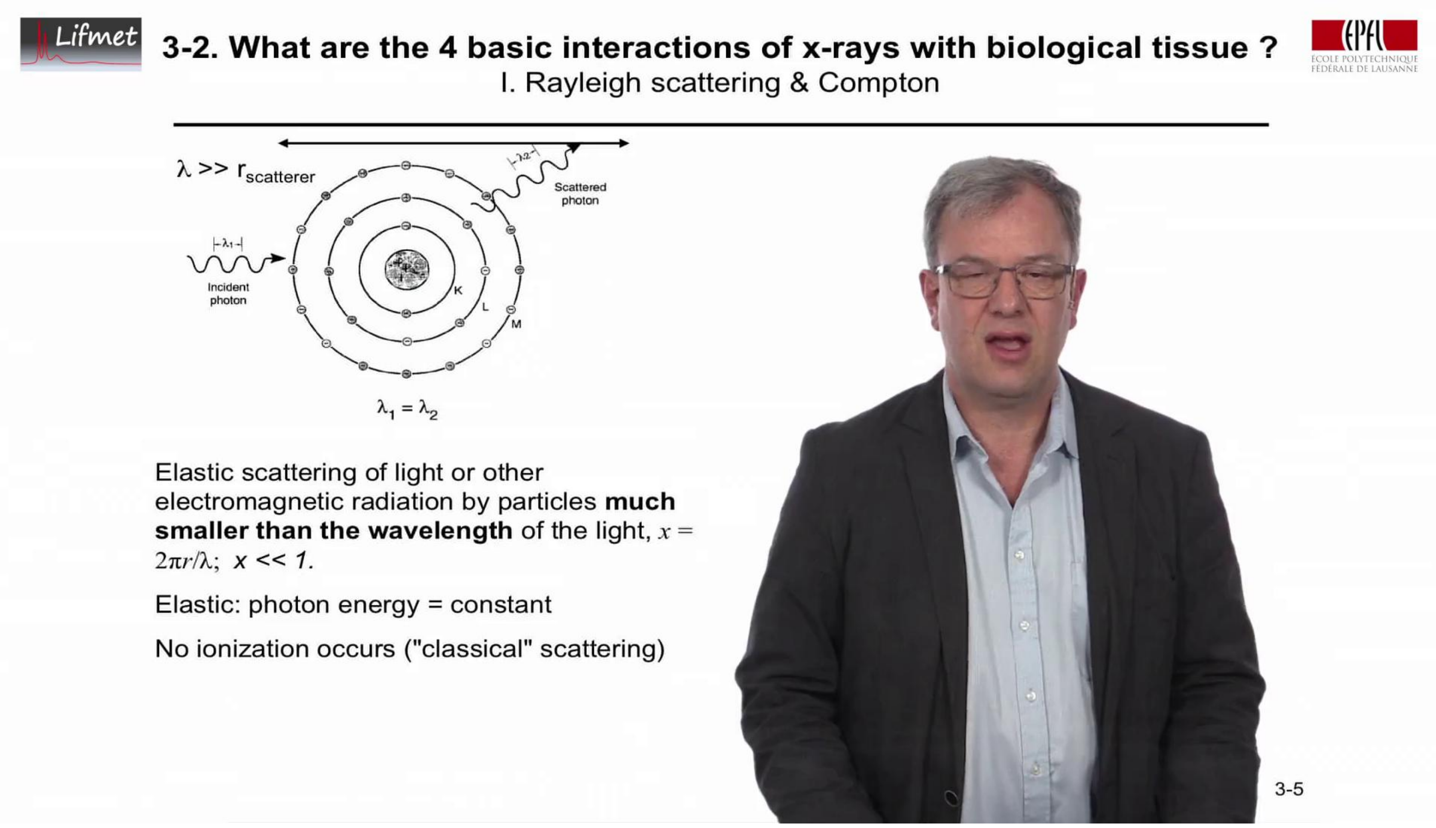
Video



So, in the next two sections, I'd like to discuss the four fundamental interactions of x-rays with biological tissue that are to be considered. Let's start out with Rayleigh scattering. In Rayleigh scattering, this happens when the light interacts with particles whose dimensions are much smaller than the wavelength of the light. So, the dimension of the object, x, is 2πr, roughly, divided by λ if you express it, so the dimension of the object 2πr, with respect to the wavelength we get this dimensionless parameter x, and for Rayleigh scattering to occur, this parameter x has to be much smaller than 1. So in other words, the wavelength has to be much bigger than the dimension or the radius of the scattering particle that is present. We have an elastic scattering so the photon energy is constant. We have the incident photon with a wavelength λ1. It is being scattered by the scatterer. It has the wavelength λ2, and for Rayleigh scattering the condition is that λ1 is equal to λ2. In this case, since the photon does not change the wavelength it does not change the energy, and therefore it is non-ionizing. It is classical scattering of a wave with a particle.

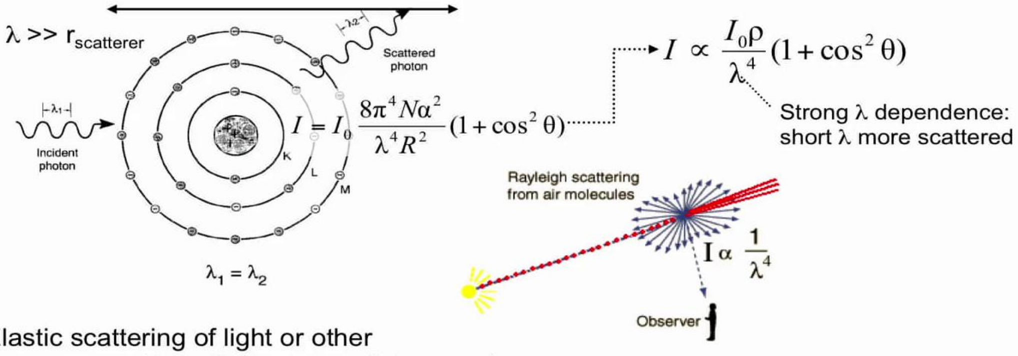
Notes

Summary

0m 04s

3-2. What are the 4 basic interactions of x-rays with biological tissue ?

I. Rayleigh scattering & Compton



Elastic scattering of light or other electromagnetic radiation by particles **much smaller than the wavelength** of the light, $x = 2\pi r/\lambda$; $x << 1$.

Elastic: photon energy = constant

No ionization occurs ("classical" scattering)

3-5

Now, the equation that I'm interested here to discuss is this one, and I'll just keep it provided here without the derivation. This says that the intensity of the scattered light is equal to the incident light, times some number of constants, and what we're really interested here is this term 1/λ to the power of 4. So we'll just look at that. So the intensity is proportional to the density of the scattering particles ρ divided by λ to the power of 4. So what this means is, the shorter the wavelength of the light as it is being scattered, the higher is the intensity of the scattered light. So, it means that infrared light is less scattered and ultraviolet light is more scattered. Or red light is less scattered, blue light is more scattered. And so, what the observer, if here's the source, say the sun, we have particles in here that scatter the light. Then the observer sees the intensity here more for the short wavelength of the scattered light, that is, blue. And this is essentially the reason why we see blue sky. Whenever you look at the sky, you see Rayleigh scattering. That's why our sky is blue. The sunlight is pretty much white when it comes in, but the sky that we see is blue because that is scattering in the atmosphere of the sunlight.

Notes

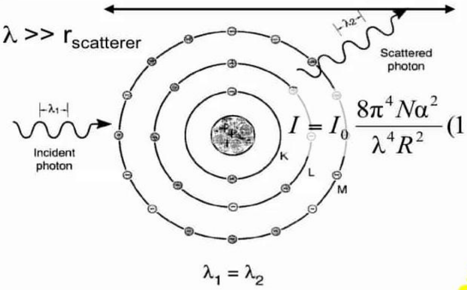
Summary

1m 41s

3-2. What are the 4 basic interactions of x-rays with biological tissue ?

I. Rayleigh scattering & Compton

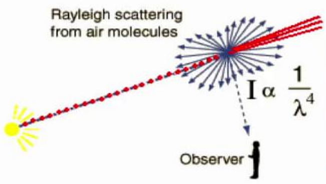
$\lambda >> r_{scatterer}$



$$I = I_0 \frac{8\pi^4 N\alpha^2}{\lambda^4 R^2}(1+\cos^2\theta)$$

$$I \propto \frac{I_0 \rho}{\lambda^4}(1+\cos^2\theta)$$

Strong λ dependence: short λ more scattered



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Elastic: photon energy = constant

No ionization occurs ("classical" scattering)

Blue sky, red sunset (smog, humidity)

Rayleigh and cloud-mediated scattering contribute to diffuse light

3-5

Here's an illustration, you can actually reproduce the Rayleigh scattering at home. What you see in this video is as you fill the bathtub with water-- water, if you look at water in a glass, it's clear, it has no color. But as you fill the bathtub with water, as you can see in this video, the color of the water turns increasingly blue. And this is because the light that is incident on the water is being scattered, and since the scattering occurs intensity is proportional to 1 over the wavelength to the fourth power, we see increasingly the short wavelength light that is... it goes towards the blue we don't see as much of the red and therefore it changes color. This is also the reason why the lakes, when we look at lakes from outside they are blue. Again, water itself has no color, but when we look at the scattering that is a lake with sufficient depth, then we see that the water takes a blueish color. That is purely an effect of Rayleigh scattering. Another effect is, like I mentioned, the blue sky, but also the red sunset. So if you have a red sunset, it means that there are scattering particles in the air. This is typically either humidity or smog.

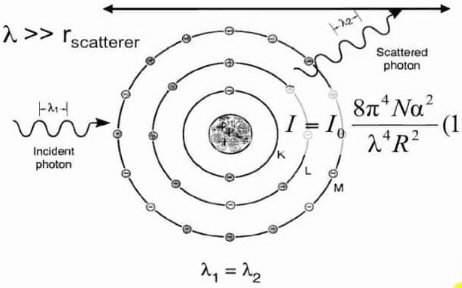
Notes

Summary

3m 14s

3-2. What are the 4 basic interactions of x-rays with biological tissue ?

I. Rayleigh scattering & Compton



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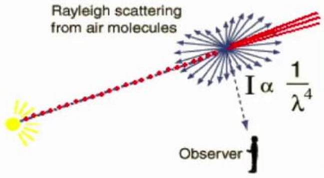

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No ionization occurs ("classical" scattering)

Blue sky, red sunset (smog, humidity)

Rayleigh and cloud-mediated scattering contribute to diffuse light

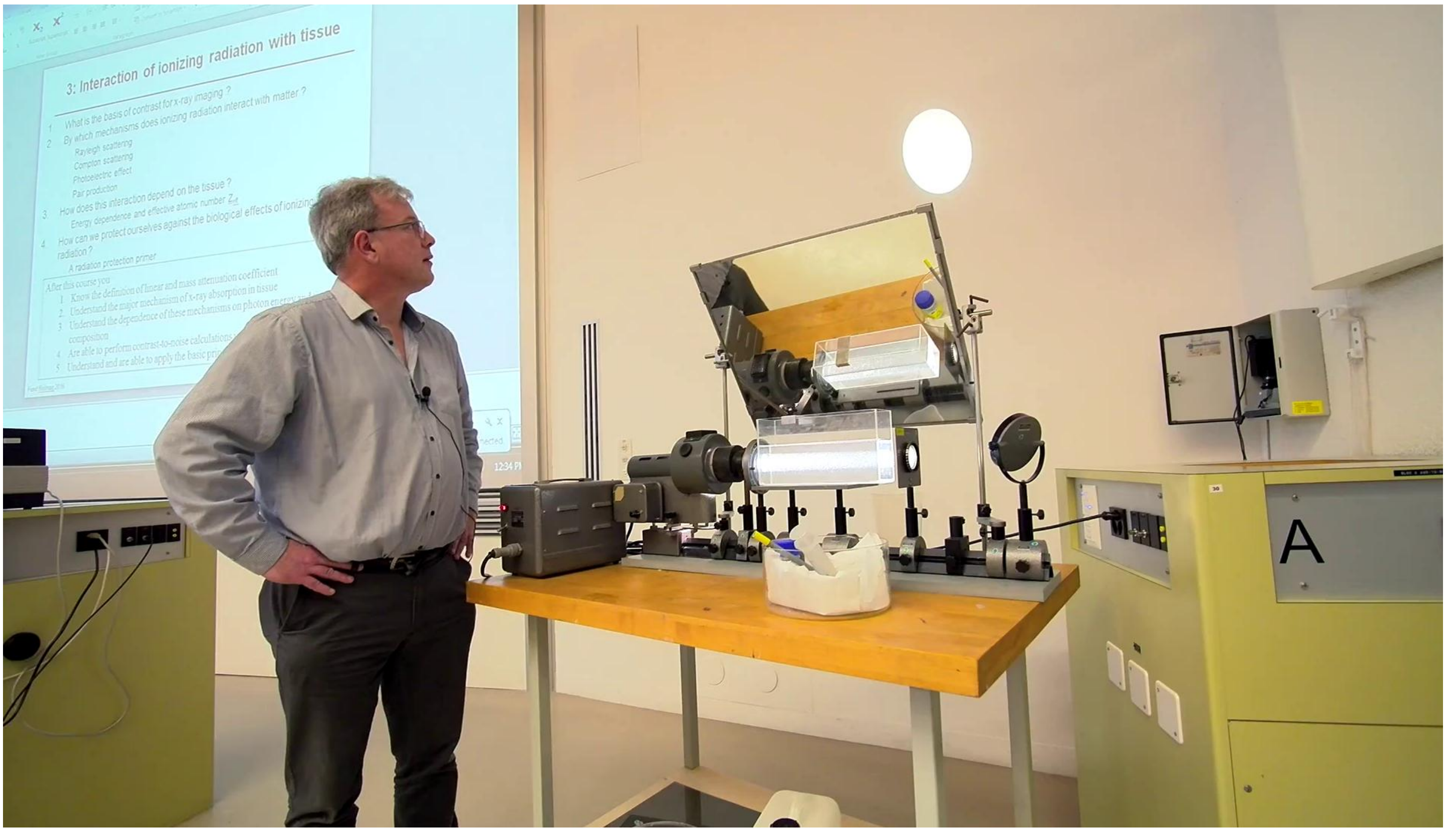
3-5

So, you have in areas where you have pollution but not too strong pollution, then you can have beautiful red sunsets. The red sunsets are a manifestation of the smog that's in the air, or if you have humidity in the air, so just before a thunderstorm for example, or before it starts to rain and the air is humid, then you also do get very colorful, very red sunsets. So this is an example here of a picture and the experiment that we will be showing shortly is illustrating in the classroom setting the effect of Rayleigh scattering demonstrated with an experimental setup.

Notes

Summary

4m 37s

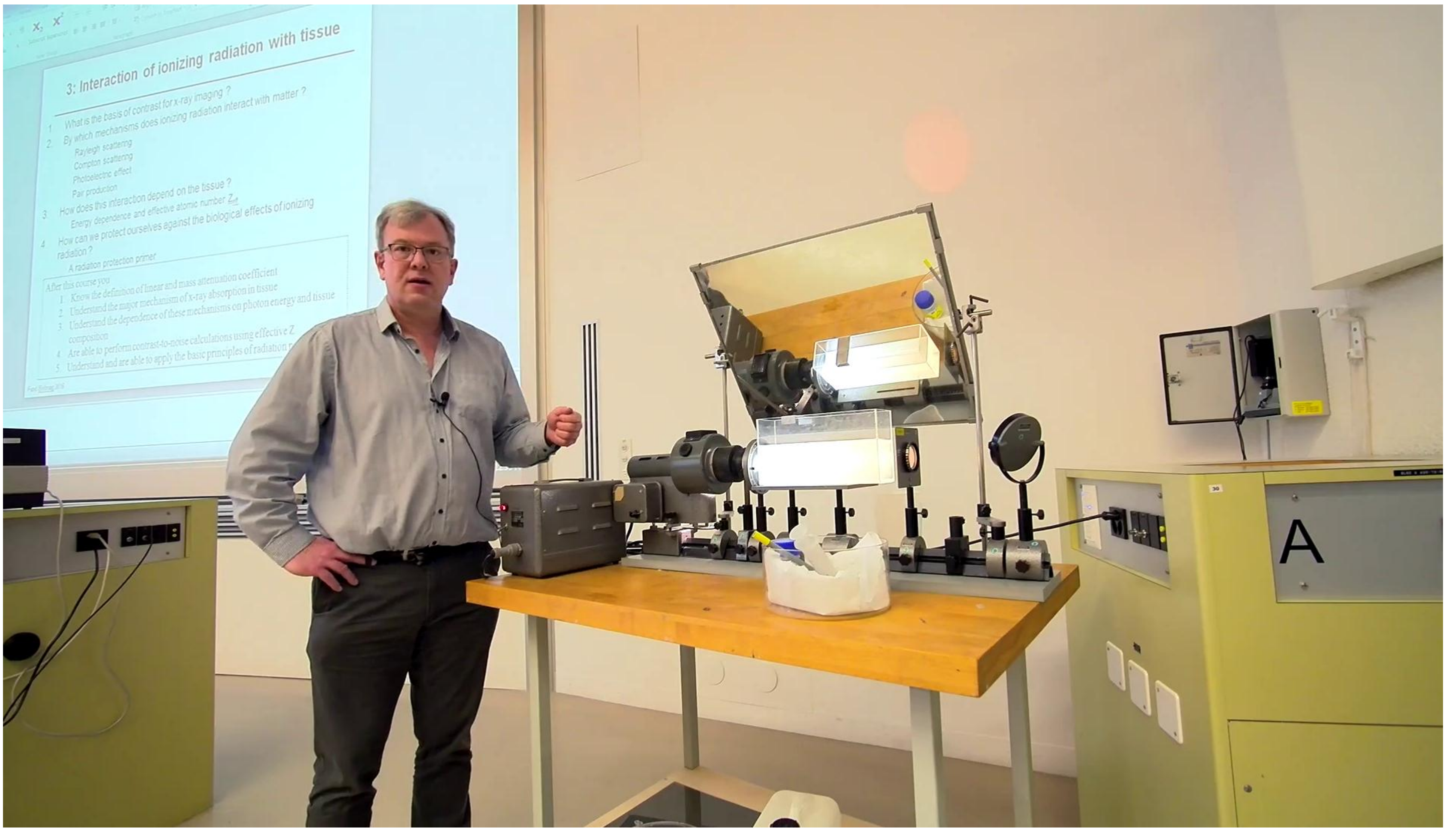


In this experiment, we will demonstrate the Rayleigh scattering, the effect that we see on moist or polluted evenings, the red sunset. So what we have here is water, I will inject into it an alcohol. This will create colloid particles with this mixture in here and start blocking out the short wavelength light and only the long wavelength light will pass through here, will be reflected by the mirror, and we should see the reflection there of the light on the wall turn gradually into red. So what I will do is now I will inject the solution. This is chemistry, so we stir well. And now we'll just have to wait for the colloids to form. And this will take a while. So now we have the mixture in there. And we'll focus on here. We should start to see it gradually change color. Just like a sunset. At the same time, as only the red light passes through, we will be seeing this turn blueish before it becomes so thick that we won't see anything, and that is basically simulating the situation as we have it in some of the bigger cities in, say, China where the pollution is very thick. Now we're starting to see it's turning slightly reddish.

Notes

Summary

5m 29s



On here, it has started changing the color, and at the same time if you look here now you see this has become a little bit blue here. Clearly it is becoming blue, just like our sky because the red light, the long wavelength has been filtered out. Now it's increasing, now we can see very clearly the changing color up here, so only the long wavelength light goes through, the short wavelength light is being scattered by Rayleigh scatttering and you see how it simultaneously is turning here increasingly blueish. That means the red light is passing through and it's only the blue light that is scattering. And now we have a really nice sunset here. It's getting more and more red. And we're seeing here the blue light. So now that's typical of a sunset with very moist air or high pollution in the air. The sun is setting very red, we have the red light here. So now you have the demonstration in classroom here of the Rayleigh scattering effect that explains to us why the sunsets are, in moist air, are red in color as well as sometimes the moon when we catch it early in the morning.

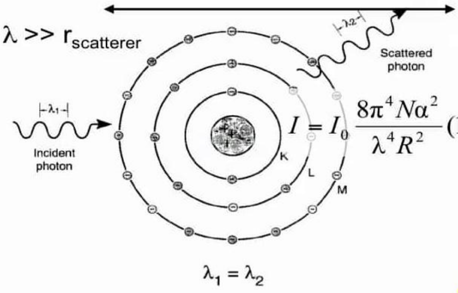
Notes

Summary

7m 27s

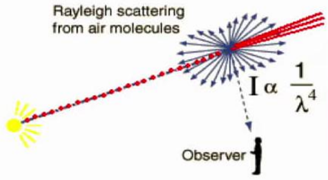
3-2. What are the 4 basic interactions of x-rays with biological tissue ?

I. Rayleigh scattering & Compton



$I = I_0 \frac{8\pi^4 N\alpha^2}{\lambda^4 R^2}(1+\cos^2\theta)$ $\rightarrow$ $I \propto \frac{I_0\rho}{\lambda^4}(1+\cos^2\theta)$

Strong λ dependence: short λ more scattered



Elastic scattering of light or other electromagnetic radiation by particles **much smaller than the wavelength** of the light, $x = 2\pi r/\lambda$; $x << 1$.

Elastic: photon energy = constant

No ionization occurs ("classical" scattering)

Scattered X-rays ⇒ deleterious effect on image quality

very low probability (< 5%) in x-ray imaging

Blue sky, red sunset (smog, humidity)

Rayleigh and cloud-mediated scattering contribute to diffuse light

E(keV)=1.2/λ(nm)

λ=1.2/E = 0.012nm (@100keV)

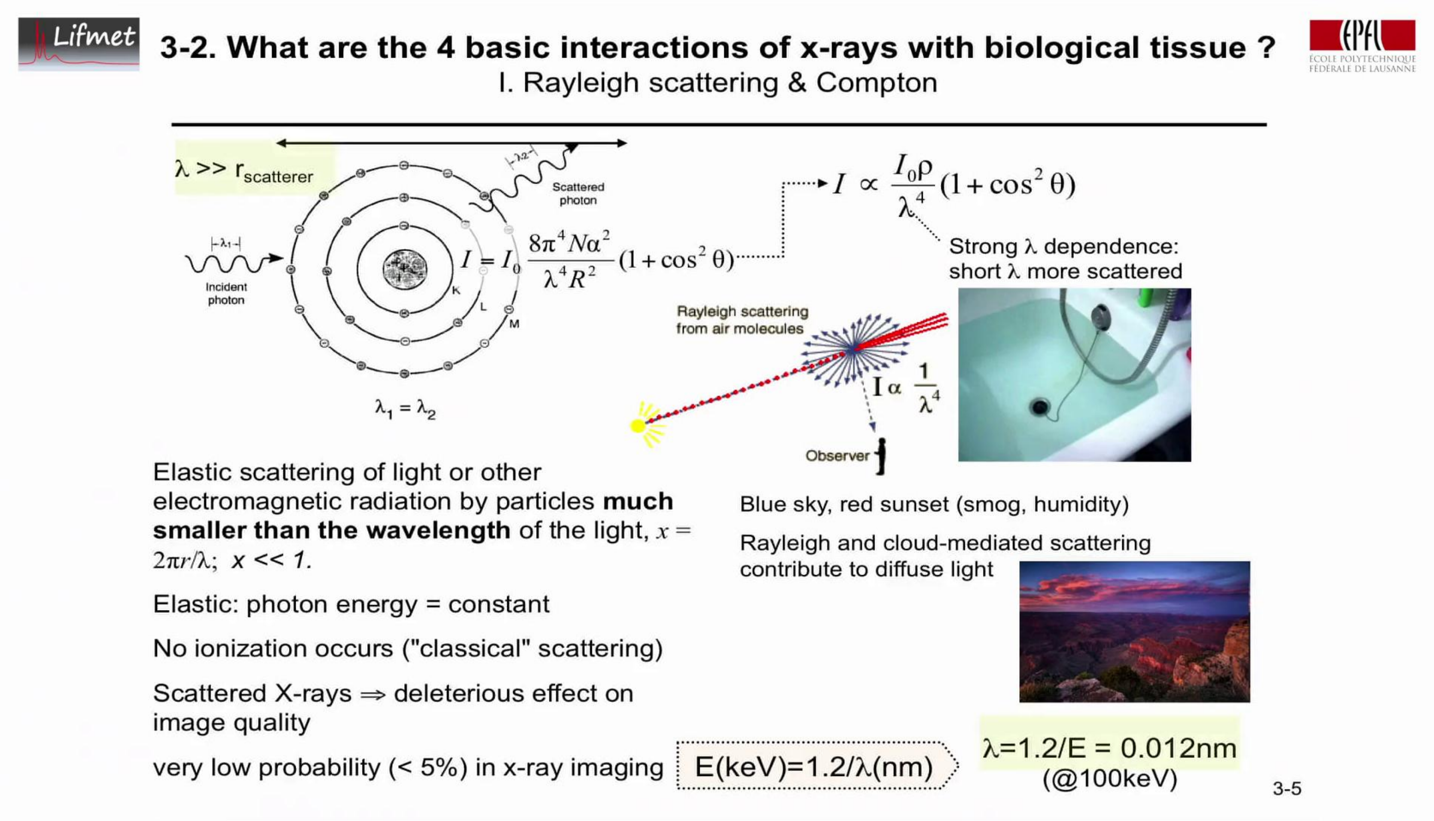
3-5

So, now if we have Rayleigh scattering, as we will see the scattered photons are not good for imaging. They have a deleterious effect on image quality. They are not good as the light that is being scattered, the x-rays. Now fortunately for x-ray imaging, Rayleigh scattering is really not an important effect. It has a low probability of occurring in x-ray imaging. Now, why is this so? Remember we had the requirement that the wavelength must be longer than the dimensions of the scatterer. And we have introduced last week the relationship that we calculate the energy in kiloelectron volts (keV). It's equal to 1.2/λ, which is the wavelength in nanometers. So we can, from the photon energy, calculate the wavelength with this expression, and this is what we are doing here. So we'll take for 100 keV, we can calculate, therefore, the wavelength, and we'll find that the wavelength for a 100 keV photon is 0.12 nanometers, or it's a tenth of an angstrom. Now remember, we had the requirement that the wavelength of the x-ray should be large compared to the dimensions of the scatterer.

Notes

Summary

8m 56s

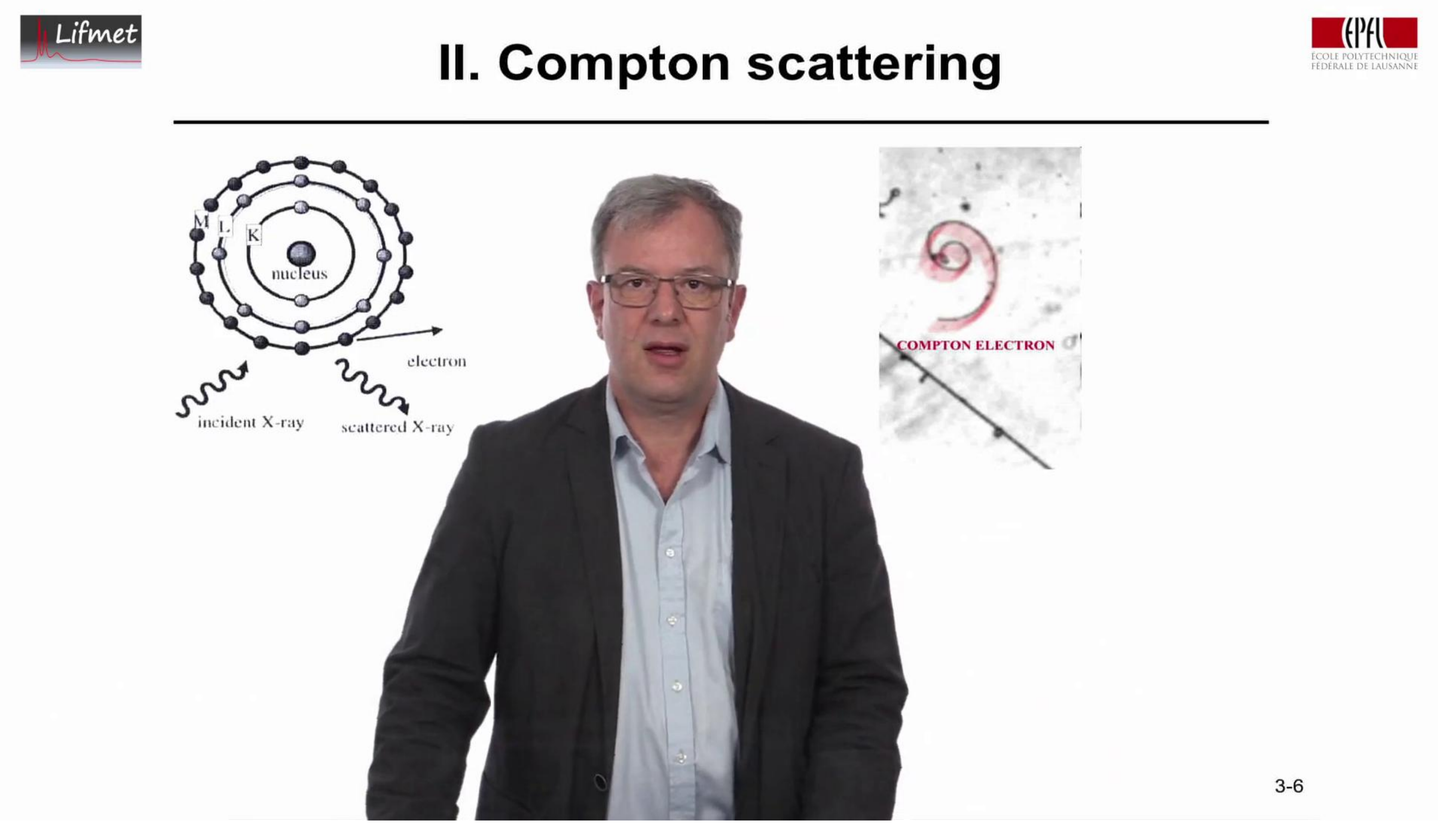


Now atoms, the dimensions of atoms are on the order of angstrom, and we are already here at a wavelength that is a fraction of an angstrom, and therefore we do not have scatterers that satisfy at this high photon energy the requirement for Rayleigh scattering to occur, and, therefore, it's probability is very low, and we can for the most part ignore its existence in the x-ray imaging. If we take for example, instead of 100 keV we take 10 keV x-rays, then we're getting a wavelength of 1.2 angstrom, 0.12 nanometers. This is still very small compared to the size of most molecules that we encounter.

Notes

Summary

10m 14s

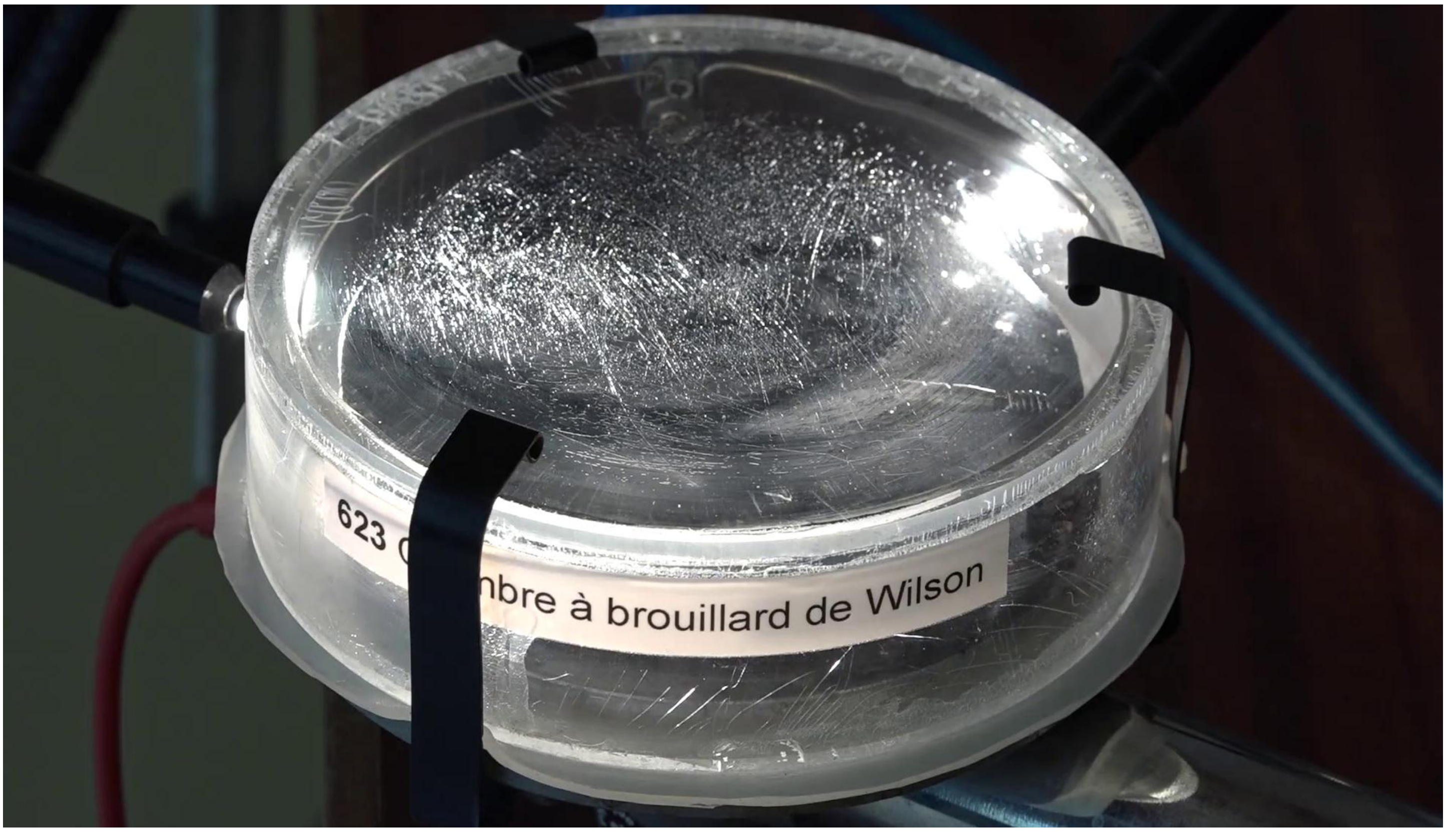


Now, I'd like to go on with the second effect and that's Compton scattering. Compton scattering is a very important effect, and we're going to spend a little bit more time discussing the Compton effect. What happens in Compton in this type of scattering? We have an incident x-ray, that x-ray interacts with an outer shell electron. This outer shell electron is then ejected, and a scattered x-ray is being produced. Here's an example of an experiment that illustrates the Compton effect. We have an x-ray that is incident, and then an electron is being produced, a free electron, it's ionizing. This is measured in the magnetic field so the electron undergoes a spiral trajectory until it comes to rest. This is a cloud chamber demonstration of the electron that's being produced.

Notes

Summary

11m 08s



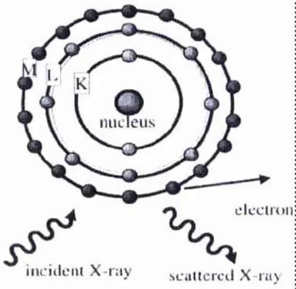
Okay, so here we have a chamber with saturated water vapor. We have here the source of the radioactivity. These are beta particles. And when I depress the tension in this chamber, we will see the condensation of the electrons as they pass through the saturated water vapor. Three, two, one...

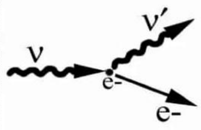
Notes

Summary

12m 04s

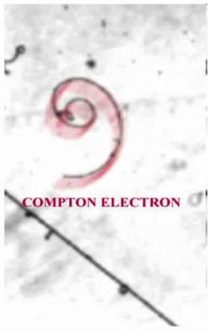
II. Compton scattering





$\nu' < \nu$

Electron is initially at rest
e- gains energy
E binding energy is neglected (why?)



Arthur Holly Compton
Physics, 1927

Occurs at the outer shell electrons

⇒ ionization

Scattered photon: subject to subsequent interactions
(Rayleigh, Compton scattering or photoelectric effect)

3-6

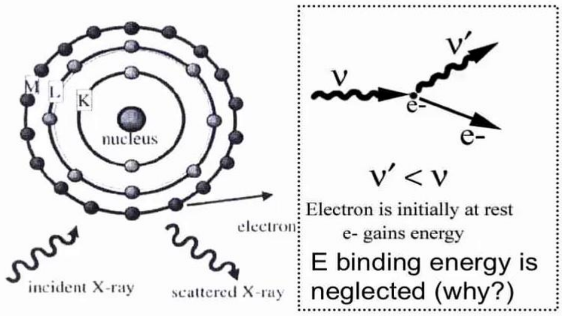
Okay, to come back to Compton scattering, it occurs at the outer shell electrons. It leads to ionization, producing an ionized atom and an electron. So, what we are really looking at the process here is we have an incident photon, we have an electron, we have a scattered electron, μ prime, and we have the ejected electron. Well, suppose that the electron is addressed initially and it gains energy through this scattering process. Now this process can be viewed as a simple collision. And for this collision, we will neglect the binding energy of the outer shell electron. Why can we do that? Well, consider we have an incident photon with an energy of 100 keV, and we have a binding shell energy of an outer shell electron of say 2.5 keV. This is two and a half percent of the energy of the photon, so we'll make a very good approximation assuming that we can neglect the binding energy of the outer shell electron. Compton effect is named after Sir Arthur Holly Compton. He received for his discovery of this effect named after him, the Nobel Prize in Physics in 1927. So, we produce a scattered photon. The scattered photon has subsequent interactions in the tissue. These can be what we've discussed.

Notes

Summary

12m 32s

II. Compton scattering



Arthur Holly Compton
Physics, 1927

Occurs at the outer shell electrons

⇒ ionization

Scattered photon: subject to subsequent interactions
(Rayleigh, Compton scattering or photoelectric effect)

Probability increases with

photon energy

electron density

Electron/mass density in tissues ~ constant (independent of Z)
→ proportional to the **density of the material**

3-6

These can be, then, Rayleigh scattering; they can be subsequent Compton events, Compton scattering or, which is what we'll discuss after the Compton effect, photoelectric effects. So, there are subsequent interactions for this photon. In general, the probability for Compton scattering to occur, it increases with photon energy, and it increases with electron density. The second one makes sense, as the electron density increases there are more outer shell electrons present in the tissue so the probability of the x-ray interacting with an outer shell electron increases. Now, typically in tissues we have the electron-to-mass density is constant, is pretty much constant, and so the electron density is largely independent of the atomic number. And so the Compton effect is pretty much proportional to the density of the material.

Notes

Summary

14m 07s

Relativistic linear momentum

a brief tour back to 1st year physics

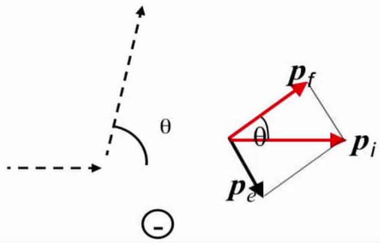
From the definition p≡mv (which is true at any velocity) it follows

$$\boxed{\vec{p} = m(v)\vec{v} = m_0 \frac{\vec{v}}{\sqrt{1-v^2/c^2}}}$$

$$\vec{p}\cdot\vec{p} = m_0^{\ 2}\frac{\vec{v}\cdot\vec{v}}{1-v^2/c^2}$$

The value of p is

$$E^2 - m_0^{\ 2}c^4 =$$



3-7

So, we are going to take you back to a brief tour of first-year physics. Most of you probably have had some special relativity in your first-year physics courses or later. Basically the phenomenon that we are observing here is we have a photon that imparts the electron. The electron gains momentum, and the photon is deviated here by this angle θ. So the incident photon hits the electron here. The electron is ejected in a certain distance, and the photon is deflected by this angle θ. And so, we will take now the definition of linear momentum in its relativistic form. So linear momentum is mass times velocity. The mass here is the rest mass divided by the square root of this factor. That's the relativistic mass times the velocity. That's the relativistic expression for linear momentum. We can now calculate the magnitude of linear momentum, that is, take the scalar product of linear momentum with itself. We get this expression here on the right. That's just multiplying this with itself. And then we can actually show that the value of *p*, that is the norm of *p*, is equal to *E* squared minus *m0 c* squared, *m0* squared, *c* to the power of four.

Notes

Summary

15m 15s

Relativistic linear momentum

a brief tour back to 1st year physics

From the definition p≡mv (which is true at any velocity) it follows

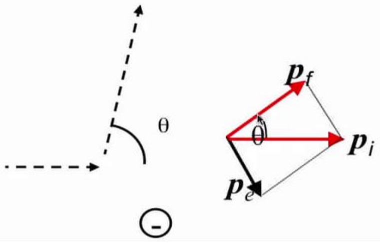
$$\boxed{\vec{p} = m(v)\vec{v} = m_0 \frac{\vec{v}}{\sqrt{1-v^2/c^2}}}$$

$$\vec{p}\cdot\vec{p} = m_0^{\,2}\frac{\vec{v}\cdot\vec{v}}{1-v^2/c^2}$$

The value of p is

$$E^2 - m_0^{\,2}c^4 = \underbrace{\overbrace{m_0^{\,2}\frac{c^4}{1-v^2/c^2}}^{\text{Relativistic kinetic energy } E=m(v)c^2} - m_0^{\,2}\frac{c^4\left(1-v^2/c^2\right)}{1-v^2/c^2}}_{}$$

$$m_0^{\,2}\frac{v^2}{1-v^2/c^2}c^2 = (pc)^2$$



3-7

We'll take the relativistic energy, that's *m0*, that's this term, that's the relativistic kinetic energy. Mass as a function of velocity, that's this term here times *c* squared, that's the famous formula of this gentleman here, E=mc2 and we'll take this term here, which is added here and when we do the calculation here, then we find that this term here so we've taken this term here minus this. If we replaced *E* squared with *mc* squared so that's *mc* to the power of four. *m* is here, that's this term squared. We'll take this term here, which is this term here, but we'll have made the denominator the same. And then we obtain, we can simplify this, and we'll find that this term here is equal to the linear momentum times the speed of light squared. So to summarize, we have a vector problem incident linear momentum of the photon. We have the linear momentum of the electron afterwards and the linear momentum of the photon afterwards and here the scattering of the light of the photon happens by the angle θ. So to conclude this, we have a relationship between *p* squared that is, the value of *p* in squared is equal to this term here.

Notes

Summary

16m 39s

Relativistic linear momentum

a brief tour back to 1st year physics

From the definition p≡mv (which is true at any velocity) it follows

$$\boxed{\vec{p} = m(v)\vec{v} = m_0 \frac{\vec{v}}{\sqrt{1-v^2/c^2}}}$$

$$\vec{p}\cdot\vec{p} = m_0^{\,2}\frac{\vec{v}\cdot\vec{v}}{1-v^2/c^2}$$

The value of p is

Relativistic kinetic energy $E=m(v)c^2$

$$E^2 - m_0^{\,2}c^4 = \underbrace{\overbrace{m_0^{\,2}\frac{c^4}{1-v^2/c^2}} - m_0^{\,2}\frac{c^4\left(1-v^2/c^2\right)}{1-v^2/c^2}}$$

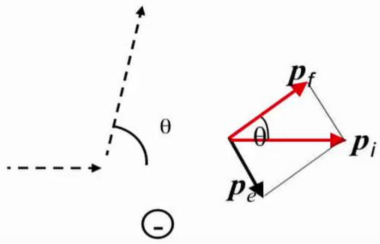
$$m_0^{\,2}\frac{v^2}{1-v^2/c^2}c^2 = (pc)^2$$

$$\boxed{\vec{p}\cdot\vec{p} = \frac{E^2}{c^2} - m_0^{\,2}c^2}$$

NB. Light carries energy, but moves at the speed of light (!) c.

Photon with energy E:
particle with rest mass (m_0=0)
(otherwise its energy would be infinite, since v=c) :

$$\boxed{\left|\vec{p}\right| = \frac{E}{c}}$$



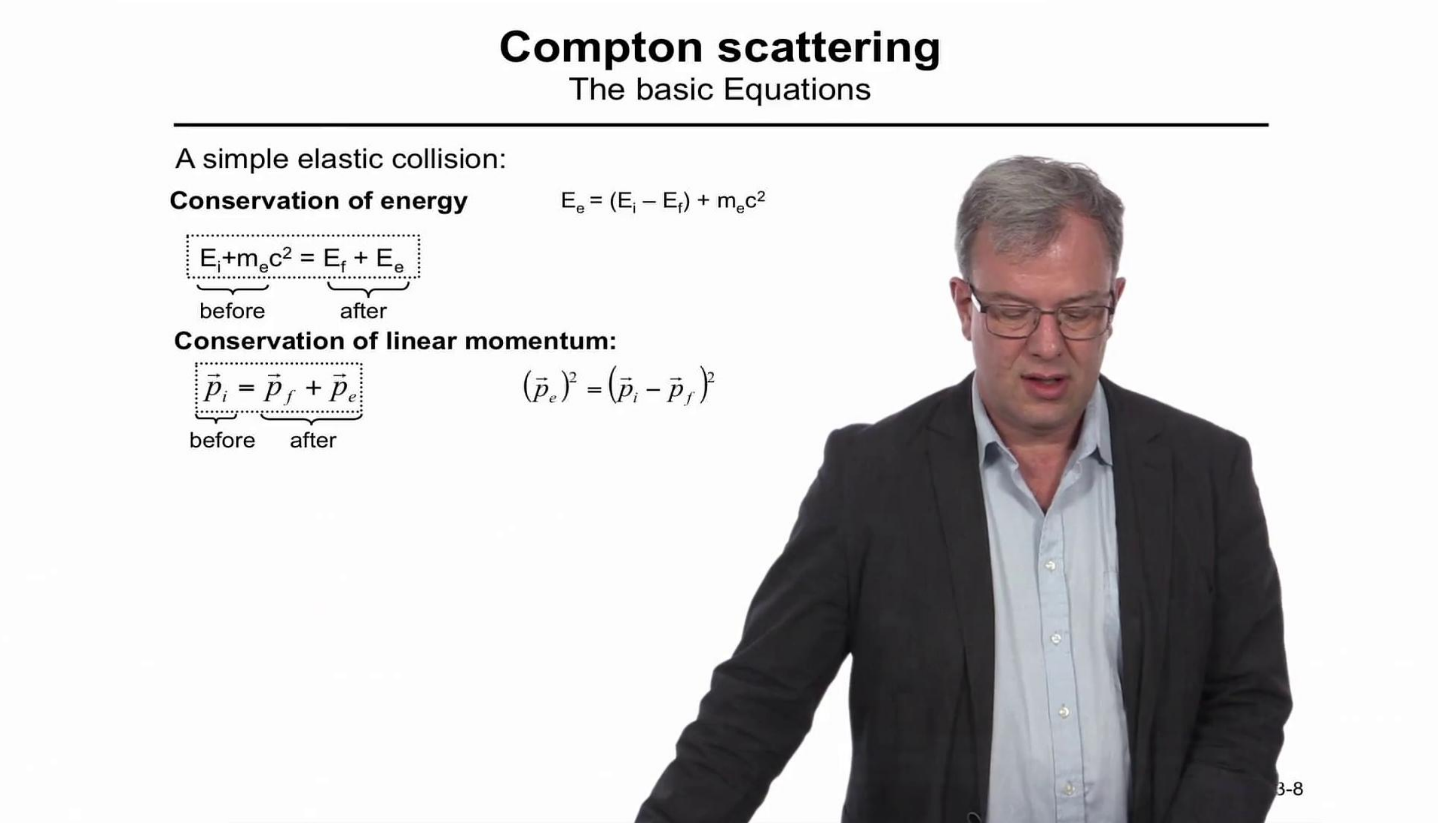
3-7

Now, there are some interesting consequences. We know that light carries energy. If we go outside we warm up in the sunlight so, obviously, the light that comes from the sun carries energy. Yet, light moves at, by definition, by the speed of light. So, if we look at this expression here, for the mass, if the light had a mass, then it would have at the speed of light, infinite mass, so this means that the photon with an energy *E*, is a particle with rest mass equal to zero. So we have a rest mass *m0* equals zero otherwise we would end up here with an infinite linear momentum, and its energy would be infinite since the speed of light particles equals the speed of light. So what follows from this, with this general equation here it follows that the linear momentum of a photon is equal to its energy divided by the speed of light. And we are going to use this relationship in the following to derive the Compton relationship that is so essential for x-ray imaging techniques.

Notes

Summary

18m 12s



So, if we are looking at Compton scattering, it is actually, as I pointed out, it is a simple elastic condition. We have conservation of energy, that is, the energy of the photon, the incident photon, the initial energy of the photon, the rest energy of the electron, *m e c* squared is equal to the final energy of the photon and the total energy of the electron. This is before, and this is after. So, I will now take the term and isolate the energy of the electron after the collision, so we'll isolate this term here, that is, we'll just move *Ef* to the other side and we'll have this term here. Now we also, since it's a collision, we have conservation of linear momentum. It's an isolated system. Initially, the only momentum that we have is that of the incident photon. And then, after the collision, the final momentum of the photon and the momentum of the electron. So this is before, and this is after. And here, again, I will isolate the term linked to the electron after the collision, *p e*, and we'll square it so *p e* squared equals the initial photon energy, photon momentum, minus the final photon momentum squared.

Notes

Summary

19m 26s

Compton scattering

The basic Equations

A simple elastic collision:

Conservation of energy

$$\underbrace{E_i + m_e c^2}_{\text{before}} = \underbrace{E_f + E_e}_{\text{after}}$$

$$E_e = (E_i - E_f) + m_e c^2$$

$$p_e^{\,2} = \frac{E_e^{\,2}}{c^2} - m_e^{\,2} c^2$$

Conservation of linear momentum:

$$\underbrace{\vec{p}_i}_{\text{before}} = \underbrace{\vec{p}_f + \vec{p}_e}_{\text{after}}$$

$$(\vec{p}_e)^2 = (\vec{p}_i - \vec{p}_f)^2$$

$$\boxed{E_f = \frac{E_i}{(1-\cos\theta)E_i / m_e c^2 + 1}}$$

With $\lambda = c/\nu$ $E = hc/\lambda$ it follows:

3-8

And now we'll use the relationship that *p e* squared is equal to the energy of the electron squared, divided by *c* squared minus *m e* squared *c* squared, which is what we just derived on the previous slide. We'll use these two equations. We have *p e* squared, we have an expression for that. We have the energy of the electron afterwards. Also, we have an expression for the two, we'll plug it in to this equation and solve for the energy of the scattered photon, the final energy. So what we see in this relationship is that the final energy of the photon depends on the initial energy of the photon but also depends on the angle θ. And this is what's important here is... if we know the incident energy of the photon, *Ei*, this term here, and we know by what angle the photon... and we know the energy of the photon. We can measure the energy of the photon that is detected. From that relationship we can say something about by what angle the photon has been scattered. And this is used in the x-ray techniques. I want to now also show another version of the Compton formula, that is, if we use the definition for wavelength and for the energy here, we replace now the energies with wavelength, then we get the so-called Compton shift.

Notes

Summary

20m 49s

Compton scattering

The basic Equations

A simple elastic collision:

Conservation of energy

$$\underbrace{E_i + m_e c^2}_{\text{before}} = \underbrace{E_f + E_e}_{\text{after}}$$

$E_e = (E_i - E_f) + m_e c^2$

$$p_e^2 = \frac{E_e^2}{c^2} - m_e^2 c^2$$

Conservation of linear momentum:

$$\underbrace{\vec{p}_i}_{\text{before}} = \underbrace{\vec{p}_f + \vec{p}_e}_{\text{after}}$$

$$(\vec{p}_e)^2 = (\vec{p}_i - \vec{p}_f)^2$$

$$\boxed{E_f = \frac{E_i}{(1-\cos\theta)E_i / m_e c^2 + 1}}$$

With $\lambda = c/\nu$ $E = hc/\lambda$ it follows:

$$\boxed{\lambda_f - \lambda_i = \frac{h}{m_e c}(1-\cos\theta)}$$

E_f is

Maximal if $\theta=0$: $E_f^{max} = E_i$

Minimal, if $\theta=\pi$: $E_f^{min} = \dfrac{E_i}{2E_i / m_e c^2 + 1}$

3-8

That is the shift in wavelength, the final wavelength of the scattered photon, minus the wavelength of the initial photon is equal to the number of fundamental constants, Planck's constant, the mass of the electron, speed of light times 1 minus cosine θ, that is the angle by which the photon has been scattered. So, let's look at some relationships. The final energy is maximal if θ is equal to zero. We can verify that; then we get the denominator here. This term is zero, and, therefore, the final energy of the photon after scattering is equal to the initial energy. It's as if the electron doesn't exist. That is logical, because if it doesn't scatter, then it does not lose any energy. The energy of the photon is minimal if θ is equal to π, that is, it's back-scattered. The photon moves, hits the electron, and comes back in the same trajectory so the scattering angle is 180 degrees, or equal to π, and then we can calculate for this case the minimum energy of the photon after scattering event is given by this expression is simply having cosine θ equal to -1, then we get the term 2 here. That's this term. So these are two particular cases.

Notes

Summary

22m 16s

Compton scattering

The basic Equations

A simple elastic collision:

Conservation of energy

$$\boxed{\underbrace{E_i + m_e c^2}_{\text{before}} = \underbrace{E_f + E_e}_{\text{after}}}$$

$E_e = (E_i - E_f) + m_e c^2$

$$p_e^{\,2} = \frac{E_e^{\,2}}{c^2} - m_e^{\,2} c^2$$

Conservation of linear momentum:

$$\boxed{\underbrace{\vec{p}_i}_{\text{before}} = \underbrace{\vec{p}_f + \vec{p}_e}_{\text{after}}}$$

$$(\vec{p}_e)^2 = (\vec{p}_i - \vec{p}_f)^2$$

$$\boxed{E_f = \frac{E_i}{(1-\cos\theta)E_i / m_e c^2 + 1}}$$

With $\lambda = c/\nu$ $E = hc/\lambda$ it follows:

$$\boxed{\lambda_f - \lambda_i = \frac{h}{m_e c}(1-\cos\theta)}$$

E_f is

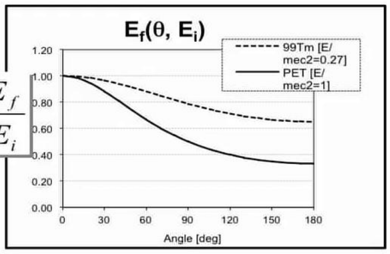
Maximal if $\theta = 0$: $E_f^{max} = E_i$

Minimal, if $\theta = \pi$: $E_f^{\ min} = \dfrac{E_i}{2E_i / m_e c^2 + 1}$

$E_i = m_e c^2$:

$$E_f = \frac{m_e c^2}{(2-\cos\theta)}$$

$\dfrac{E_f}{E_i}$



3-8

One case that is of interest, as we will see in three weeks is if the energy of the photon that comes in is equal to the rest energy of the electron, *m e c* squared. That's for positron emission tomography, and, in this case, if *Ei* over *m e c* squared is 1, then we have *m e c* squared here for the incident energy. That's we're just replacing this relationship here, and we have in the denominator 2 minus cosine θ. So let's look now, on the y axis, we have the final energy, the scattered energy divided by the incident energy on the y axis as a function of the angle here now in degrees, and we'll have now two popular cases. One is metastable technetium, very often used in medical imaging. Here the energy relative to the rest energy of the electron is equal to 0.027. That's this case here, and for positron emission tomography, that is this case here, we have the energy of the photon is equal to *m e c* squared, we have this relationship that is seen here, so it's a stronger dependence. Now what is the critical element here? The critical element here is we have... the effect depends on the photon energies implicated, so it depends on the isotope that's being used, or the x-ray that's being used.

Notes

Summary

23m 42s

Compton scattering

The basic Equations

A simple elastic collision:

Conservation of energy

$$E_e = (E_i - E_f) + m_e c^2$$

$$\underbrace{E_i + m_e c^2}_{\text{before}} = \underbrace{E_f + E_e}_{\text{after}}$$

$$p_e^{\,2} = \frac{E_e^{\,2}}{c^2} - m_e^{\,2} c^2$$

$$E_f = \frac{E_i}{(1-\cos\theta)E_i / m_e c^2 + 1}$$

Conservation of linear momentum:

$$\underbrace{\vec{p}_i}_{\text{before}} = \underbrace{\vec{p}_f + \vec{p}_e}_{\text{after}}$$

$$(\vec{p}_e)^2 = (\vec{p}_i - \vec{p}_f)^2$$

With $\lambda = c/\nu$ $E = hc/\lambda$ it follows:

$$\lambda_f - \lambda_i = \frac{h}{m_e c}(1-\cos\theta)$$

E_f is

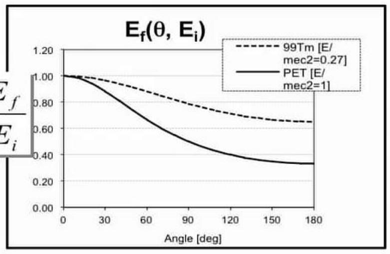
Maximal if $\theta=0$: $E_f^{max} = E_i$

Minimal, if $\theta=\pi$: $E_f^{\,min} = \frac{E_i}{2E_i / m_e c^2 + 1}$

$E_i = m_e c^2$:

$$E_f = \frac{m_e c^2}{(2-\cos\theta)}$$

$\frac{E_f}{E_i}$



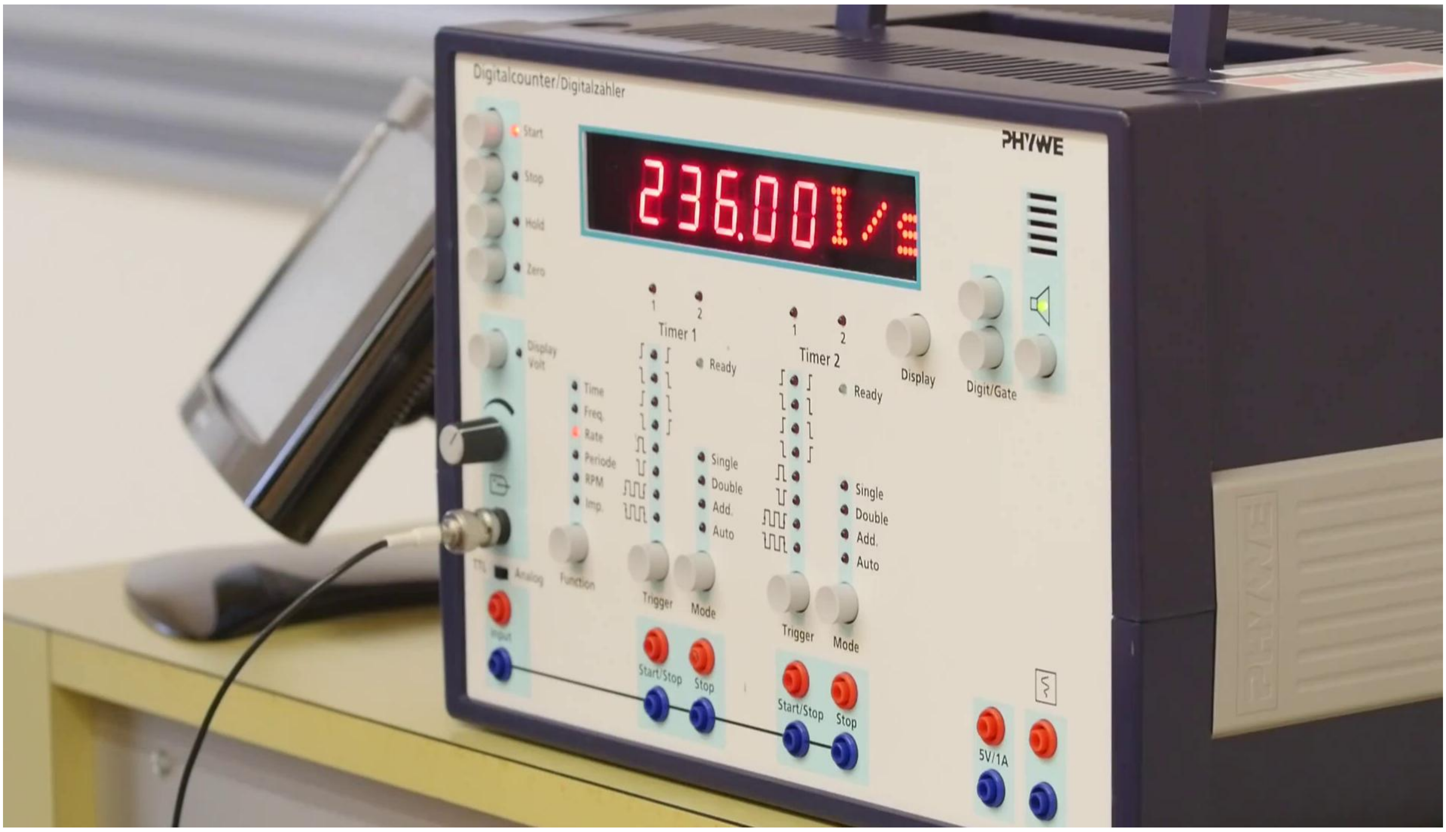
3-8

But also, what is crucial here is, if we can calculate the final energy relative to the initial energy, if we can measure that, then we can say, well, this photon has been scattered by 90 degrees. And if it's been scattered by 90 degrees and we detect it then it's not a good photon, it's been scattered too much and it's likely to represent an effect in an area of the tissue where we don't want to measure, and so based on this energy discrimination we can eliminate heavily scattered photons.

Notes

Summary

25m 06s



Okay, so what we have here is a demonstration of the Compton effect. We have the radioactive source here, that's the x-ray tube which we've discussed last week. How the x-rays are being produced in the tube. There's a little plastic element in the middle. That serves as a scatterer, and the detector is here. So at this point we have a strong deflection angle. And what we are measuring and what you're hearing is the energy of the photons detected. So we can see now at this big angle of deflection we have a count of 260. The number is not important what it means. And as I now reduce the angle between the beam which is emitted in this direction, we can see that this gradually increases as the angle gets closer to zero degrees that is, the photon energy increases we have the scatterer here and as we are going close to zero degree we have a very high count, so very high photon energy in this direction. I'll increase the angle again, away from zero and we can see how as I gradually increase the angle the photon energy is being reduced.

Notes

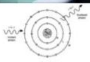
Summary

25m 38s

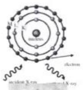
Credits

Don Smith Photography
http://donsmithphotography.aminus3.com/image/2008-05-29.html

Rayleigh scattering and light refraction *(video extract)*
https://youtu.be/Jg7gpCuLkrA



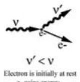
"The Essential Physics of Medical Imaging"
Fig. 3-6, from Jerrold T. Bushberg, J. Anthony Seibert, Edwin M. Leidholt, John M. Boone



"Introduction to biomedical imaging"
Fig. 1-7, from Andrew Webb

The Compton Effect
http://teachers.web.cern.ch/teachers/archiv/hst2002/bubblech/mbitu/electromag-events1.htm

Arthur Holly Compton
http://science.nasa.gov/science-news/science-at-nasa/1999/features/ast20apr99_1/



Compton Scattering
http://venables.asu.edu/quant/proj/compton.html

Albert Einstein, The Nobel Foundation
http://www.nobelprize.org/nobel_prizes/physics/laureates/1921/einstein-facts.html

Notes

Summary

26m 51s